The Sectoral Origins of the Spending Multiplier^{*}

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Abstract

The aggregate spending multiplier crucially depends on the sectoral origin of government purchases. To establish this result, we characterize analytically the response of aggregate output to sector-specific government spending shocks in a tractable production-network economy. The response is larger when government spending originates in sectors with a relatively small contribution to private final demand, low markup, high labor intensity, and in those located downstream in the supply chain. We confirm these predictions and evaluate their quantitative relevance within a calibrated multi-sector model of the U.S. economy that embeds several dimensions of sectoral heterogeneity. We also provide suggestive empirical evidence supporting them. Based on the calibrated model, we document significant dispersion in the aggregate spending multiplier associated with sectoral government purchases. Finally, we illustrate how differences in the sectoral composition of purchases across U.S. government levels (i.e., general, federal, and state & local) lead to sizable differences in the spending multiplier. The latter ranges from 0.47 for federal defense spending, which is mainly concentrated in manufacturing, to 0.82 for state & local spending, which is mostly oriented towards services.

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1 Introduction

Government purchases of goods and services from the private sector are heterogeneously distributed across highly diverse industries.¹ This basic observation is largely overlooked by theoretical studies of fiscal policy, which tend to condense the entire economy into a representative sector, and lump government purchases from the different industries into a single aggregate. By construction, the onesector framework cannot address the role of sectoral characteristics and inter-sectoral linkages in the transmission of government spending shocks. This limitation becomes even more severe when assessing large stimulus plans, which typically target specific industries.

This paper takes a bottom-up approach to study the aggregate implications of sectoral government purchases. A large literature has examined the role of sectoral heterogeneity and production networks in shaping aggregate fluctuations caused by idiosyncratic/sectoral disturbances.² Surprisingly, however, little is known about how the aggregate effects of sector-specific government spending shocks map into the intrinsic characteristics of the sector being shocked and its position in the supply chain. We tackle this question both analytically and quantitatively, offering novel insights: the aggregate spending multiplier is larger when government spending originates in sectors with a relatively small contribution to private final demand, low markup, high labor intensity, and in those located down-stream in the production network. We also provide suggestive evidence supporting these predictions.

Our analytical insights are based on a stylized two-sector Cobb-Douglas economy, in which production is carried out using labor and intermediate inputs, and prices are set by imposing a markup over marginal costs. The two sectors differ in their contribution to private final demand (i.e., their sectoral consumption share), markup, labor intensity, and position in the production network, one sector being located upstream and the other downstream. To transparently illustrate how these characteristics shape the aggregate effects of sectoral spending shocks, we assume that labor supply is infinitely elastic. As a result, consumption is proportional to the real wage in equilibrium.³ Furthermore, we allow labor to be imperfectly mobile across the two sectors, thus enabling relative prices to vary in response to demand shocks.⁴

By construction, the response of aggregate output to a sectoral spending shock comprises a direct effect, which we assume to be symmetric across sectors,⁵ and a general-equilibrium effect. We show that

¹The standard deviation of sectoral government purchases is 45 percent larger than that of sectoral value added. The correlation between the sectoral contribution to government purchases and sectoral value added is 0.39. These figures are based on information from the 2018 U.S. Input-Output Tables at the 3-digit level of the North American Industry Classification System (NAICS). Furthermore, using information on the universe of procurement contracts by the U.S. federal government, Cox et al. (2021) document that government spending is granular, being concentrated among few firms and sectors, and that its sectoral allocation differs from that of private spending.

²E.g., Long and Plosser (1983), Horvath (1998, 2000), Foerster, Sarte and Watson (2011), Gabaix (2011), Acemoglu et al. (2012), Carvalho (2014), di Giovanni, Levchenko and Mejean (2014), Atalay (2017), and Bigio and La'O (2020).

 $^{^{3}}$ The assumption of an infinitely elastic labor supply neutralizes the resource-constraint effect associated with government spending shocks. This assumption is relaxed in the quantitative analysis.

⁴Under perfect labor mobility and constant returns to scale in production, relative prices are independent of the demand side of the economy, and are therefore unresponsive to government spending shocks (see also Acemoglu, Akcigit and Kerr, 2016). Imperfect labor mobility, however, is not the only technical expedient to generate variation in relative prices. For instance, one could alternatively assume that the production technology exhibits decreasing returns to scale.

 $^{^{5}}$ Our primary focus is not to explain the difference in the aggregate effects of sectoral spending shocks induced mechanically by heterogeneity in the sectors' exposure to their own shocks, but instead to relate those effects to the

the latter involves the product of two statistics pertaining to the sector being shocked: the response of its relative price, and a loading factor that depends (i) positively on the distance between the sector's steady-state contribution to *total* and *private* final demand, and (ii) negatively on the distortion in the sector's relative size — measured by its steady-state employment share — arising from markup pricing.

An immediate implication of this result is that the aggregate effects of sectoral spending shocks are independent of their origin when relative prices do not adjust. The underlying intuition rests on the fact that the real wage can — under constant returns to scale — be expressed as a weighted average of sectoral relative prices, where the weights correspond to the sectors' contribution to total final demand, adjusted for their size distortion. Absent relative-price adjustment, the real wage remains constant, and so does private consumption. When relative prices adjust, instead, consumption is diverted away from the more expensive good and towards the cheaper one. This expenditure switching, however, does not give rise to any change in aggregate income (again, implying that the real wage remains constant) if the weight attached to the sectors' relative prices coincide (or vary proportionally) with their consumption shares.⁶ Under this knife-edge condition — which is attained when the economy is efficient and both sectors contribute equally to private and total final demand — the loading factor becomes nil.

In the more general case where the weights in the wage equation are not proportional to the consumption shares, changes in relative prices generate an income effect whose magnitude depends on the sectoral origin of government purchases, thereby implying that the sectoral composition of government demand matters for aggregate outcomes. Specifically, the response of aggregate value added to a spending shock decreases with the consumption share of the shocked sector. Intuitively, holding constant a sector's share in public spending, its contribution to total final demand falls less than proportionally when its consumption share declines. As a result, the increase in the relative price of the shocked sector receives a weight larger than its consumption share, leading to a larger response of the real wage. Hence, everything else constant, aggregate output rises more when the government demands more goods from the industry that contributes less to private consumption.

Furthermore, under markup pricing, the response of aggregate value added is larger when spending originates in the less distorted sector, *ceteris paribus*. Underlying this result is the fact that the weight attached to the relative price of a given sector in the wage expression decreases with that sector's size distortion. When the two sectors are otherwise identical, sectoral prices respond with opposite signs but with the same magnitude, such that the response of the real wage (and aggregate output) to a sectoral spending shock is larger the smaller the distortion of the shocked sector.

In our economy, sectoral heterogeneity in the size distortion stems from differences in the markup, labor intensity, or position in the network. When the two sectors are otherwise symmetric, a spending shock originating in the sector with a smaller markup or with a higher labor intensity gives rise to a smaller labor wedge (relative to the efficient allocation), leading to a larger response of aggregate

⁽non-policy) structural characteristics of the shocked sectors.

⁶When the weights associated with sectoral relative prices are equal to the consumption shares, the nominal wage coincides with the nominal price index, becoming effectively the numeraire, and implying that the real wage is constant. In this case, changes in relative prices cause consumers' budget line to rotate, but not to shift.

output. Moreover, when the two sectors only differ in their position in the network, aggregate output rises more when spending originates in the downstream sector. Intuitively, under equal markups and labor intensities, the upstream sector is more distorted than the downstream sector because it provides all the intermediate inputs used in the economy, and these are subject to double marginalization, as they are marked up both when they are produced and when they are used.

We then ask whether these findings carry over to a richer and more realistic setting. To do so, we develop a quantitative multi-sector model that features multiple sources of sectoral heterogeneity and a complex production network, and calibrate it to 57 industries of the U.S. economy. The model's predictions corroborate those of the stylized economy: The aggregate value-added multiplier is larger when spending shocks originate in sectors with a relatively small contribution to private final demand, low markup, high labor intensity, and in those located downstream in the production network. In contrast, sectoral heterogeneity in the degree of price rigidity is found to be less important in explaining differences in the aggregate multiplier of sectoral shocks.

Performing a direct empirical test of these predictions is challenging because standard approaches to identify sectoral government spending shocks in panel settings only enable one to estimate their crossindustry effects, the aggregate general-equilibrium effects being subsumed in the time dummies.⁷ Thus, we resort to an indirect approach that exploits heterogeneity in the sectoral composition of (federal) defense purchases across U.S. states. More specifically, we use the universe of contracts awarded by the U.S. Department of Defense to pin down the sectoral content of defense spending within each state. We use these data to run a regression that estimates the average local multiplier — defined as the dollar response in the GDP of a given state following a dollar increase in defense spending in that state — as well as its interaction with a term that represents the extent to which spending is concentrated in sectors with a given characteristic. The estimates corroborate our predictions: the local multiplier tends to be larger when the government purchases from a given state are more skewed towards sectors that contribute relatively less to private final demand, have lower markups, are labor intensive, and are located downstream in the supply chain.

The mapping between the aggregate effects of government purchases and the sectoral characteristics of their industry of origin implies that the aggregate spending multiplier is asymmetric across sectoral spending shocks and that, by extension, it varies with the sectoral allocation of government purchases. We leverage the fully fledged structure of the calibrated model to offer quantitative insights into these two implications.

We find significant dispersion in the size of the aggregate multiplier of U.S. sectoral government purchases, which ranges from 0.23 to 1.13. Overall, the output response is lower when the government buys from upstream industries with a relatively large contribution to investment and low labor intensity, like manufacturing. On the other hand, it is larger when the government raises its demand for goods produced by downstream labor-intensive industries, such as retail trade and educational and health-care services.

To illustrate the quantitative relevance of the sectoral composition of spending for the aggregate multiplier, we exploit the differences in the sectoral content of purchases across layers of the U.S.

⁷See, for instance, Nakamura and Steinsson (2014) and Chodorow-Reich (2019, 2020).

government — general, federal (defense and non-defense), and state & local.⁸ While the aggregate value-added multiplier associated with spending by the general government amounts to 0.72, this figure masks substantial heterogeneity across government levels, with values ranging from 0.47 for federal defense spending to 0.82 for spending by the state & local government. Likewise, we find significant differences in the aggregate consumption multiplier, which is negative (-0.17) for defense spending, but positive (0.13) for state & local government purchases. This variance is due to the fact that defense spending is mainly concentrated in upstream industries with low labor intensity, whereas state & local spending is mostly oriented towards downstream labor-intensive sectors. Importantly, these findings contribute to explaining the large dispersion in the empirical estimates of the effects of government spending: studies that rely on federal defense spending tend to report small output multipliers and a crowding-out of consumption (e.g., Barro and Redlick, 2011; Ramey, 2011); instead, measuring spending at the general-government level typically leads to large output multipliers and a crowding-in (e.g., Blanchard and Perotti, 2002; Auerbach and Gorodnichenko, 2012).

Related literature Our paper builds on the large literature that studies the spending multiplier, focusing predominantly on government consumption in the form of purchases of goods and services from the private sector (see Ramey, 2019 for a recent survey). A rapidly growing branch of this literature examines the effects of government purchases within multi-sector production-network economies.⁹ In a companion paper (Bouakez, Rachedi and Santoro, 2023), we show that input-output interactions and sectoral heterogeneity in price rigidity amplify the output effects of aggregate spending shocks relative to the one-sector framework, taking as given the sectoral composition of government purchases. Proebsting (2022) emphasizes the combination of imperfect labor mobility and demand segmentation as an alternative amplification mechanism. Acemoglu, Akcigit and Kerr (2016) examine the propagation of sectoral spending shocks via the production network, but their framework inhibits relative-price adjustment, implying that inter-sectoral linkages are irrelevant for the aggregate spending multiplier. Devereux, Gente and Yu (2023) build on Acemoglu, Akcigit and Kerr (2016) to study the cross-country spillovers of government spending shocks through international production networks. Dong and Wen (2019) compare the transmission of money injections via the production network with that of sectoral technology and government spending shocks.

Closer to our paper, Baqaee and Farhi (2019), Cox et al. (2021), and Flynn, Patterson and Sturm (2021) study environments in which the sectoral composition of government purchases matters in the aggregate. Baqaee and Farhi (2019) and Flynn, Patterson and Sturm (2021) consider economies in which workers with different marginal propensities to consume are heterogeneously distributed across industries,¹⁰ while Cox et al. (2021) lay out a model with sectoral bias in the allocation of government purchases and heterogeneity in pricing frictions across industries. We share with these papers the idea

⁸We study otherwise identical versions of the model that only differ in the sectoral composition of government spending. More specifically, we consider different linear combinations of sectoral spending shocks, where the weights replicate the sectoral composition of purchases at each U.S. government level.

⁹The first work on the effects government spending within a multi-sector model is Ramey and Shapiro (1998), which shows that costly capital reallocation across sectors alters the spending multiplier relative to the one-sector framework. This paper, however, abstracts from production networks.

¹⁰Relatedly, Guerrieri et al. (2022) show how the interplay between households with different marginal propensities to consume and input-output linkages affects the effectiveness of fiscal policy in responding to supply shocks.

that the spending multiplier depends on the sectoral composition of public purchases.¹¹ However, we provide a novel perspective on what could be driving this dependence, by mapping the aggregate effects of sectoral government spending into various primitives of the recipient industry. We believe we are the first to offer this insight, both analytically and quantitatively.

Our paper is also related to recent research that examines the macroeconomic implications of microeconomic shocks in inefficient production-network economies. Existing papers in this literature, however, have focused on sectoral productivity and markup shocks, showing, in particular, how sectoral distortions affect aggregate efficiency in the presence of input-output linkages (e.g., Jones, 2013; Liu, 2019; Baqaee and Farhi, 2020; Bigio and La'O, 2020).

Finally, this paper connects to a body of applied work that studies highly disaggregated public purchases and estimates their effects at the industry/firm level (e.g., Nekarda and Ramey, 2011; Acemoglu, Akcigit and Kerr, 2016; Slavtchev and Wiederhold, 2016; Auerbach, Gorodnichenko and Murphy, 2020; Kim and Nguyen, 2020; Hebous and Zimmermann, 2021; Coviello et al., 2022). These studies typically adopt identification strategies that difference out the aggregate general-equilibrium effects of sectoral shocks, thus preventing a clear mapping between the estimated cross-industry multipliers of government purchases and their aggregate counterparts. Our paper complements this strand of the literature by developing a structural framework that takes into account the general-equilibrium interactions shaping the aggregate effects of sector-specific government spending shocks.

Structure of the paper The rest of the paper is organized as follows. In Section 2, we study both analytically and quantitatively how the aggregate effects of sectoral government purchases depend on the characteristics of the industry from which those purchases are made. We also provide suggestive evidence supporting the theoretical predictions. In Section 3, we measure the dispersion in the aggregate multipliers associated with U.S. sectoral government spending shocks, and compare the aggregate multipliers implied by the sectoral composition of purchases across the different levels of the U.S. government. Section 4 concludes.

2 Sectoral Characteristics and the Aggregate Effects of Sectoral Government Spending

The purpose of this section is to relate the aggregate output effects of sectoral government spending shocks to the attributes of the sector in which those shocks originate, including its position in the production network. We start by characterizing this mapping analytically, in a stylized two-sector economy. We then validate our analytical results in the context of a rich, quantitative multi-sector model, calibrated to the U.S. economy. Finally, we provide suggestive evidence supporting our theoretical predictions.

¹¹In doing so, we complement earlier work that shows that the effects of government spending are not invariant to considerations such as the financing scheme (e.g., Leeper, Plante and Traum, 2010), the stance of monetary policy (e.g., Christiano, Eichenbaum and Rebelo, 2011), and the state of the economy (e.g., Auerbach and Gorodnichenko, 2012).

2.1 A stylized model

Consider a flexible-price economy with two inter-connected sectors that use labor and intermediate inputs to produce, and sell goods both to consumers and to the government. Firms in each sector produce differentiated varieties and operate under monopolistic competition. The two sectors differ along four structural dimensions: (i) their contribution to private final demand, as they have different shares in aggregate consumption, (ii) their elasticity of substitution across varieties, such that they have different steady-state markups, (iii) their labor intensity, and (iv) their position in the production network. To capture the latter feature in a tractable and parsimonious way, we consider an upstream sector, denoted by u, which supplies all the intermediate inputs used by both sectors, and a downstream sector, denoted by d, which demands intermediate inputs, but provides none. The model allows for imperfect labor mobility across sectors, and nests the limiting case in which labor is perfectly mobile. Because the model has no endogenous state variable, it can be solved period by period. Thus, we drop the time subscript in the remainder of this section.

2.1.1 The environment

Households The representative household has a utility function that is logarithmic in consumption, C, and linear in total labor, N:

$$u(C, N) = \log C - \theta N, \qquad \theta > 0.$$
⁽¹⁾

The linearity of the utility function with respect to labor means that the latter is indivisible, as in Hansen (1985), which in turn implies that the Frisch elasticity of labor supply is infinite. This assumption, which we relax in Section 2.2, is convenient because it neutralizes the resource-constraint effect associated with changes in government spending, thereby greatly simplifying the algebra.

To allow for imperfect labor mobility across sectors, we follow Huffman and Wynne (1999), Horvath (2000), and Bouakez, Cardia and Ruge-Murcia (2009), and posit that the total amount of labor provided by the household is a CES function of the labor supplied to each sector, that is,¹²

$$N = \left[\omega_{N,u}^{-\frac{1}{\nu_N}} N_u^{\frac{1+\nu_N}{\nu_N}} + \omega_{N,d}^{-\frac{1}{\nu_N}} N_d^{\frac{1+\nu_N}{\nu_N}}\right]^{\frac{\nu_N}{1+\nu_N}},\tag{2}$$

where $\omega_{N,u} + \omega_{N,d} = 1$, with $\omega_{N,s}$ being the weight attached to labor provided to sector s (s = u, d), and $\nu_N \geq 0$ is (the absolute value of) the elasticity of substitution of labor across sectors. This specification nests the special case in which $\nu_N \to \infty$, so that labor is perfectly mobile and, as a result, nominal wages are equalized across sectors. Under fully flexible prices, this also implies that sectoral relative prices are unresponsive to demand shocks. Instead, as long as $\nu_N < \infty$, labor is imperfectly mobile and both sectoral wages and relative prices can differ.¹³

 $^{^{12}}$ Note that this assumption does not contradict labor indivisibility, as one can think of the representative household as a family consisting of two members, each working in a different sector.

¹³Empirically, imperfect labor mobility across sectors is certainly a more plausible characterization of the labor market than perfect mobility. Lee and Wolpin (2006) find that labor adjusts very sluggishly in response to shocks, and is

The household's supply of labor to sector s is given by

$$N_s = \omega_{N,s} \left(\frac{\mathcal{W}_s}{\mathcal{W}}\right)^{\nu_N} N, \qquad s = u, d, \tag{3}$$

where \mathcal{W}_s denotes the nominal wage in sector s, and $\mathcal{W} = \left[\omega_{N,u}\mathcal{W}_u^{1+\nu_N} + \omega_{N,d}\mathcal{W}_d^{1+\nu_N}\right]^{\frac{1}{1+\nu_N}}$ is the aggregate nominal wage index, which satisfies $\mathcal{W}_u N_u + \mathcal{W}_d N_d = \mathcal{W}N$.

The household pays a nominal lump-sum tax, T, to the government, so that its budget constraint is

$$PC + T = \mathcal{W}N,\tag{4}$$

where P denotes the consumption-based price index.

Firms

Producers The production process is split in two stages. A continuum of monopolistically competitive producers, indexed by $j \in [0,1]$, combine labor and intermediate inputs to produce differentiated varieties of goods. These varieties are then aggregated into a single good in each sector by a representative perfectly competitive wholesaler.

Producer j in sector s has the following Cobb-Douglas production technology:

$$Z_{s}^{j} = \left(N_{s}^{j}\right)^{1-\alpha_{H,s}} \left(H_{s,u}^{j}\right)^{\alpha_{H,s}}, \quad \text{for } j \in [0,1] \text{ and } s = u, d, \tag{5}$$

where Z_s^j denotes its gross output, N_s^j denotes its use of labor, and $H_{s,u}^j$ denotes the intermediate inputs it buys from the upstream sector. The parameter $\alpha_{H,s} \in [0, 1]$ measures the gross-output-based intensity of intermediate inputs in sector s.

The representative wholesaler in sector s has the following CES aggregation technology:

$$Z_s = \left[\int_0^1 Z_s^{j \frac{\epsilon_s - 1}{\epsilon_s}} dj \right]^{\frac{\epsilon_s}{\epsilon_s - 1}}, \quad s = u, d, \tag{6}$$

where Z_s denotes total output of good s, and ϵ_s is the elasticity of substitution across varieties within sector s. Denoting by P_s^j the price set by producer j in sector s, the price of good s is then given by $P_s = \left[\int_0^1 P_s^{j1-\epsilon_s} dj\right]^{\frac{1}{1-\epsilon_s}}$. In each sector, prices are flexible and set as a constant markup, $\vartheta_s \equiv \frac{\epsilon_s}{\epsilon_s-1}$, over marginal cost.

Wholesalers' output in both sectors is sold as final goods both to a representative consumptiongood retailer and to the government. The output of the upstream sector is also sold as intermediate inputs to producers in both sectors. Imposing symmetry across producers within each sector yields

reallocated imperfectly across sectors in the short and medium run. Moreover, Krueger and Summers (1988), Gibbons and Katz (1992), and Neumuller (2015), among others, document large sectoral wage differentials. Finally, Katayama and Kim (2018) show that imperfect labor mobility provides a better account of the comovement between output and hours worked than alternative explanations based on the wealth effects associated with labor supply.

the following market-clearing conditions at the sectoral level:¹⁴

$$Z_u = C_u + G_u + H_{u,u} + H_{d,u}, (7)$$

$$Z_d = C_d + G_d, \tag{8}$$

where C_s denotes the retailer's purchase of consumption goods from sector s, and G_s denotes government purchases from sector s.

Consumption-good retailers A representative consumption-good retailer purchases goods from each sector and assembles them into a consumption bundle sold to the households. Its technology is given by

$$C = C_u^{\omega_{C,u}} C_d^{\omega_{C,d}},\tag{9}$$

where $\omega_{C,u} + \omega_{C,d} = 1$, with $\omega_{C,s}$ being the consumption share of sector s. The retailer's optimal demand for the good produced by sector s is

$$C_s = \omega_{C,s} \left(\frac{P_s}{P}\right)^{-1} C, \qquad s = u, d, \tag{10}$$

where P_s denotes the nominal price of the good produced in sector s. The zero-profit condition of the consumption-good retailer implies that $P = \omega_{C,u}^{-\omega_{C,u}} \omega_{C,d}^{-\omega_{C,u}} P_u^{\omega_{C,u}} P_d^{\omega_{C,d}}$.

Government Government purchases from the two sectors are exogenously determined and are financed through lump-sum taxes paid by the household, which implies the following budget constraint for the government:

$$P_u G_u + P_d G_d = T. (11)$$

Aggregation and auxiliary assumptions Defining $Q_s \equiv \frac{P_s}{P}$ as the real price of the good produced by sector *s*, real value added in this sector is obtained by subtracting the real cost of the intermediate inputs it uses from the real value of its gross output:

$$Y_s = Q_s Z_s - Q_u H_{s,u}, \qquad s = u, d.$$

$$\tag{12}$$

By consolidating the household's and government's budget constraints, one can then express aggregate real value added as

$$Y \equiv Y_u + Y_d = C + Q_u G_u + Q_d G_d. \tag{13}$$

To solve the model, we log-linearize the equilibrium conditions around a non-stochastic steady state in which, for convenience, sectoral nominal wages are constrained to be equal. This property, which we obtain by equating the sectoral weights $\omega_{N,s}$ to the steady-state employment share, $\varpi_s \equiv N_s^*/N^*$,

¹⁴These market-clearing conditions reflect the assumption that the input-output matrix has a column of ones and a column of zeros, as the upstream sector provides intermediates inputs to itself and to the downstream sector, whereas the latter provides none. This structure is the most parsimonious way to allow for differences in the sectors' position in the production network, while holding constant all the remaining sectoral attributes.

ensures that versions of the model with different degrees of labor mobility share the same steady state.¹⁵

Based on the log-linear model, we derive analytical results regarding the propagation of sectoral government spending shocks and their aggregate effects, which we relate to the characteristics of the sector being shocked and its position in the network. In the remainder of this section, lowercase variables denote percentage deviations of their uppercase counterparts from their steady-state values.

2.1.2 Analytical results

Let us introduce the following notation: $\gamma \equiv \frac{\sum_{s=u,d} Q_s^* G_s^*}{Y^*}$ denotes the steady-state share of total government spending in aggregate value added, $\omega_{G,s} \equiv \frac{Q_s^* G_s^*}{\sum_{s=u,d} Q_s^* G_s^*}$ denotes the steady-state contribution of sector *s* to total public spending, and $\mu_s \equiv \frac{Q_s^* (C_s^* + G_s^*)}{Y^*} = (1 - \gamma) \omega_{C,s} + \gamma \omega_{G,s}$ denotes its steady-state contribution to total final demand.

In Appendix A, we show that the response of aggregate value added to a spending shock in sector s is given by

$$\frac{dy}{dg_s} = \gamma \omega_{G,s} + \left[\frac{\psi_1 \left(\mu_s - \omega_{C,s}\right) - \psi_2 \left(\varpi_s^e - \varpi_s\right)}{1 - \omega_{C,s}}\right] \frac{dq_s}{dg_s}, \qquad s = u, d, \tag{14}$$

where $\psi_1 = 2 - \gamma > 0$, $\psi_2 \equiv \frac{1-\gamma}{1-\alpha_{H,d}} > 0$, and ϖ_s^e is the steady-state employment share of sector s in the absence of markup pricing (i.e., under efficiency). By definition, ϖ_s collapses to ϖ_s^e when both sectoral markups are equal to 1 (see Equations (A.25)–(A.26) in Appendix A).

The first term on the left-hand side of Equation (14) measures the direct effect of the shock, whereas the second term captures its general-equilibrium effect. The latter is characterized by the product of two statistics pertaining to the sector being shocked: the response of its relative price, and a loading factor that is (i) increasing in the distance between the sector's contribution to total final demand and its consumption share ($\mu_s - \omega_{C,s}$), and (ii) decreasing in the sector's relative size distortion measured by its steady-state employment share — arising from markup pricing ($\varpi_s^e - \varpi_s$).

In order to study the mapping between the aggregate effects of sectoral spending shocks and the non-policy characteristics of the shocked sector, we henceforth assume that the two sectors contribute equally to total public spending in the steady state, by imposing that $\omega_{G,u} = \omega_{G,d} = \frac{1}{2}$. This allows us to isolate the role of the sector's contribution to private final demand and its size distortion, *ceteris paribus.*¹⁶ We start by presenting an irrelevance result stating the general conditions under which the composition of government spending — or, alternatively, the sectoral origin of spending shocks — is irrelevant for aggregate output.

Proposition 1 An irrelevance result. The response of aggregate value added is independent of the sectoral origin of a spending shock if

(i) relative prices do not respond to the shock, or

¹⁵Steady-state variables are denoted by an asterisk. Appendix A reports the list of non-linear equilibrium conditions, the steady-state equilibrium, and the log-linearized equations.

¹⁶The assumption that the sectors contribute equally to government spending in the steady state is relaxed in the quantitative analysis carried out in Section 3.

(ii) the economy is efficient (i.e., $\vartheta_s = 1$ for s = u, d) and the sectors contribute equally to private (and total) final demand (i.e., $\omega_{C,s} = \mu_s = \frac{1}{2}$ for s = u, d).

Proof. See Appendix A. \blacksquare

In order to understand the intuition underlying parts (i) and (ii) of the proposition, notice that, under the simplifying assumption of an infinite Frisch elasticity, consumption is — up to a firstorder approximation — equal to the aggregate real wage, w. Given the constant-returns-to-scale production technology, the latter can be expressed as a weighted average of sectoral relative prices, where the weights correspond to the sectors' contribution to total final demand, adjusted for their size distortion:¹⁷

$$c = w = \frac{\overline{\omega}_u - \alpha_{H,d}}{\overline{\omega}_u^e - \alpha_{H,d}} \mu_u q_u + \frac{\overline{\omega}_d}{\overline{\omega}_d^e} \mu_d q_d.$$
(15)

Absent relative-price adjustment, aggregate consumption remains unchanged in response to spending shocks. In this case, the response of aggregate output is independent of the sectoral characteristics and the production network, as stated in part (i) of the proposition. Note that this setting, in which relative prices are unresponsive to demand shocks, is similar to that considered by Acemoglu, Akcigit and Kerr (2016), who show that sectoral spending shocks propagate upstream through the production network, from downstream sectors to their input supplying industries. Part (i) of the proposition, however, shows that this upstream propagation of sectoral shocks is entirely irrelevant for their aggregate implications.¹⁸

Changes in relative prices induce an expenditure-switching effect whereby consumers substitute the more expensive good with the cheaper one. However, when the economy is efficient (i.e., $\varpi_s = \varpi_s^e$ for s = u, d) and each of the two sectors contributes equally to total and private final demand (i.e., $\mu_s = \omega_{C,s}$ for s = u, d) — such that the loading factor in (14) becomes nil — the weights in Equation (15) collapse to the sectoral consumption shares. Under this knife-edge condition, the nominal wage coincides with the nominal price index, becoming effectively the numeraire, and implying that the real wage (and thus consumption) is constant. In this case, changes in relative prices only cause consumers' budget line to rotate, without inducing any income effect: the fall in consumption expenditure on one good is exactly offset by the increase in consumption expenditure on the other good, leaving aggregate consumption unchanged. The result in part (ii) follows immediately if the two sectors are symmetric in their contribution both to private and total demand.

In turn, this result implies that, in efficient economies, the network *per se* remains irrelevant for the aggregate effects of sectoral government spending shocks, even when relative prices adjust. Notice that this result holds globally (not only up to a first-order approximation) and does not depend on the Cobb-Douglas specification of preferences and technology.

The response of relative prices to sectoral spending shocks Given that relative-price adjustment is a necessary condition for the response of aggregate value added to depend on the sectoral

¹⁷See Appendix A.

¹⁸In Section 2.2.3, we show that the irrelevance of the network for the aggregate effects of sectoral spending shocks under constant relative prices continues to hold in a more general setting that takes into account the resource-constraint effect (i.e., in which the Frisch elasticity is finite).

origin of the shock, Proposition 2 below shows how relative prices respond to sectoral government spending shocks.

Proposition 2 Sectoral government spending shocks have no effect on relative prices if labor is perfectly mobile. Under imperfect labor mobility, an increase in government spending in a given sector raises its relative price and lowers the relative price of the other sector. That is, for s, x = u, d and $s \neq x$,

$$\begin{array}{rcl} \frac{dq_s}{dg_s} &=& \frac{dq_s}{dg_x} &=& 0 & \mbox{if } \nu_N \to \infty, \\ \frac{dq_s}{dg_s} &>& 0 & \mbox{and} & \frac{dq_s}{dg_x} &<& 0 & \mbox{otherwise.} \end{array}$$

Proof. See Appendix A. \blacksquare

Under perfect labor mobility, sectoral nominal wages are equalized across sectors. In this case, the assumptions of price flexibility and constant returns to scale in production imply that relative prices are independent of the demand side of the economy, and are therefore unresponsive to government spending shocks, as is also shown by Acemoglu, Akcigit and Kerr (2016). When labor is imperfectly mobile, instead, an increase in spending in a given sector raises its real wage and, in turn, its relative price.

Role of the sectoral consumption share Expression (14) shows that, to the extent that relative prices adjust, the consumption share of the shocked sector affects the response of aggregate output explicitly — through the loading factor — and implicitly — through the response of the sector's relative price. Propositions 3 below formally establishes the relationship between the response of aggregate output and the consumption share of the recipient sector, *ceteris paribus*. Since we know from Propositions 1 and 2 that the consumption share is irrelevant to the response of aggregate output under perfect labor mobility, we only focus on the case of imperfect labor mobility.

Proposition 3 Consider a sectoral spending shock and assume that the two sectors are otherwise identical. Under imperfect labor mobility ($\nu_N < \infty$), the response of aggregate value added is decreasing in the consumption share of the shocked sector. That is,

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \omega_{C,s}} < 0.$$

Proof. See Appendix A. \blacksquare

Holding constant a sector's share in public spending, its contribution to total final demand falls less than proportionally when its consumption share declines. As a result, the increase in the relative price of the shocked sector receives a weight larger than its consumption share, leading to a larger response of the real wage. This in turn implies that aggregate output rises more when the government demands more goods from the industry that contributes less to private consumption, *ceteris paribus*. **Role of the sectoral distortion** Expression (14) also implies that, for a given change in the relative price of the shocked sector, the response of aggregate value added decreases with the sector's size distortion. To understand this result, notice that the weight attached to the relative price of a given sector in Equation (15) decreases with that sector's size distortion. When the two sectors are otherwise identical, sectoral prices respond with opposite signs but with the same magnitude, such that — for a given change in relative prices — the response of the real wage (and aggregate output) is larger the smaller the distortion of the shocked sector.

In our economy, heterogeneity in the sectors' size distortion may stem from heterogeneity in the markup, labor intensity, or position in the network. Proposition 4 below formalizes the mapping between the response of aggregate output and each of these characteristics in the relevant case of imperfect labor mobility.

Proposition 4 Consider a sectoral spending shock and assume that the economy is inefficient and the two sectors are otherwise identical. Under imperfect labor mobility ($\nu_N < \infty$), the response of aggregate value added is:

(i) decreasing in the markup of the shocked sector. That is,

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \vartheta_s} < 0,$$

(ii) increasing in the labor intensity of the shocked sector. That is,

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \left(1 - \alpha_{H,s}\right)} > 0,$$

(iii) larger when the shock originates in the downstream sector than when it originates in the upstream sector. That is,

$$\frac{dy}{dg_d} > \frac{dy}{dg_u}$$

Proof. See Appendix A. \blacksquare

The results stated in parts (i) and (ii) are relatively easy to understand. The steady-state distortion in the employment share of sector s increases with its markup: the less elastic the demand for good s, the more firms in sector s curtail their input use, thus giving rise to a larger labor wedge (relative to the efficient allocation). On the other hand, an increase in the labor intensity of sector s implies that firms in that sector use relatively less intensively the marked-up good (i.e., intermediate inputs), which reduces the labor wedge.

The intuition behind the result in part (iii) lies in the way inefficiency affects the employment share of the shocked sector, depending on its position in the network. Under equal markups and labor intensities, the upstream sector is more distorted than the downstream sector because it provides all the intermediate inputs used in the economy, and these are subject to double marginalization, as they are marked up both when they are produced and when they are used (see also Liu, 2019).¹⁹

¹⁹Relatedly, Bigio and La'O (2020) study the aggregate implications of sectoral distortions stemming, for instance,

In sum, Proposition 4 implies that, everything else equal, raising government demand from the sector that has a smaller markup, higher labor intensity, or is located downstream in the production network leads to a lager response of aggregate output. We emphasize that these predictions are not driven by changes in the economy-wide allocative efficiency induced by the reallocation of labor across sectors outside the steady state. To see this, notice that the results stated in 4 continue to hold when sectoral labor is immobile across sectors (i.e., when $\nu_N = 0$). Under this extreme scenario, government spending shocks generate variation in aggregate labor, but not in its sectoral composition, which remains constant over time.

2.2 A calibrated multi-sector model

The stylized model considered in Section 2.1 provides sharp insights into how the response of aggregate value added to sector-specific government spending shocks depends on the intrinsic characteristics of the sector being shocked and its position in the supply chain. The analytical tractability of the stylized model, however, requires abstracting from certain features — including other dimensions of sectoral heterogeneity — that might be empirically relevant for the propagation of government spending and their aggregate effects. In this section, we develop a quantitative, highly disaggregated, multi-sector model that allows for multiple sources of sectoral heterogeneity and a complex production network.

The economy consists of S production sectors, which differ in their price rigidity, markup, factor intensities, use of intermediate inputs, and contribution to consumption, investment, and government purchases. Households accumulate physical capital, which they rent to firms, and producers face pricesetting frictions that give rise to nominal price stickiness. Both features make the model dynamic in nature.

We describe below the key distinctive features of the economy (relative to the stylized model), and refer to Appendix B for a thorough discussion of its structure. The parameter restrictions under which the multi-sector model nests exactly the one presented in Section 2.1 are specified in Appendix B.5.²⁰

2.2.1 The environement

Households We alter household preferences in three ways. First, the representative household receives utility not only from private consumption but also from the sum of government purchases from all sectors, where the two arguments enter the utility function in a non-separable manner, as in Bouakez and Rebei (2007).²¹ Second, we allow the elasticity of intertemporal substitution to be different from 1 by adopting a general CRRA function (over total consumption) rather than a logarithmic one. Third, we relax the assumption of linear disutility of labor, allowing the Frisch

from monopolistic competition. They show that these distortions result in an inefficient reallocation of resources across the input-output network, and that the aggregate effects of individual labor wedges compound as firms buy and sell from one another within the network. As a result, distortions in upstream sectors have larger effects on the aggregate labor wedge than distortions in downstream sectors.

²⁰The structure of the model is graphically summarized by Figure B.1 in Appendix B.

 $^{^{21}}$ As we show in Bouakez, Rachedi and Santoro (2023), assuming that private and public consumption spending are complements is helpful to generate aggregate spending multipliers that are in line with those reported in the empirical literature. In Appendix E, however, we show that this feature mainly exerts a level effect, as there is close-to-perfect correlation between the aggregate spending multipliers implied by the models with and without complementarity.

elasticity to be finite. We also assume that the representative household trades one-period nominal bonds. In each period, the household purchases investment goods, which increase the undepreciated stock of capital subject to convex adjustment costs.

As in the stylized model, we allow for imperfect labor mobility across sectors by assuming that total labor is given by:

$$N_{t} = \left[\sum_{s=1}^{S} \omega_{N,s}^{-\frac{1}{\nu_{N}}} N_{s,t}^{\frac{1+\nu_{N}}{\nu_{N}}}\right]^{\frac{\nu_{N}}{1+\nu_{N}}},$$
(16)

where $\sum_{s=1}^{S} \omega_{N,s} = 1$. Analogously, we assume that the total capital stock supplied by the representative household, K_t , is a CES aggregator of the capital stocks it rents to all the sectors, with $\nu_K \ge 0$ being (the absolute value of) the elasticity of substitution of capital across sectors, and $\omega_{K,s}$ denoting the weight attached to the capital provided to sector s, such that $\sum_{s=1}^{S} \omega_{K,s} = 1$. As is the case for labor, this assumption allows capital to be imperfectly mobile across sectors, consistently with its sluggish reallocation across industries over the business cycle (e.g., Lanteri, 2018).²²

Production A continuum of monopolistically competitive producers, indexed by $j \in [0, 1]$, combine labor, capital, and a bundle of intermediate inputs to produce differentiated varieties of goods. These varieties are then aggregated into a single good in each sector by a representative perfectly competitive wholesaler.

Producer j in sector s has the following Cobb-Douglas production technology:

$$Z_{s,t}^{j} = \left(N_{s,t}^{j} K_{s,t}^{j} K_{s,t}^{j}\right)^{1-\alpha_{H,s}} H_{s,t}^{j} K_{s,t}^{\alpha_{H,s}},$$
(17)

where $Z_{s,t}^{j}$ denotes gross output, and $N_{s,t}^{j}$, $K_{s,t}^{j}$, and $H_{s,t}^{j}$ denote labor, capital, and the bundle of intermediate inputs used by the producer. The parameters $\alpha_{N,s}$ and $\alpha_{H,s}$ are the value-added-based labor intensity and the gross-output-based intensity of intermediate inputs, respectively. Producers face price-setting frictions, such that they can reset their prices according to a Calvo-type mechanism, with ϕ_s being the sector-specific probability of not changing prices.

The representative wholesaler in sector s aggregates (using a CES technology with an elasticity of substitution across varieties ϵ_s) the different varieties supplied by the producers into a single final good, $Z_{s,t}$, which is then sold to consumption-good, investment-good, and intermediate-input retailers, as well as to the fiscal authority. Thus, the following market-clearing condition holds for sector s:

$$Z_{s,t} = C_{s,t} + I_{s,t} + \sum_{x=1}^{S} H_{x,s,t} + G_{s,t},$$
(18)

where $C_{s,t}$ and $I_{s,t}$ denote, respectively, the retailers' purchases of consumption and investment goods from the wholesaler of sector s, $H_{x,s,t}$ denotes the intermediate inputs produced by sector s and used in the production of sector x, and $G_{s,t}$ denotes government purchases from sector s.

 $^{^{22}}$ Miranda-Pinto and Young (2019) show that a model with imperfect mobility of capital across industries can fit well both the volatility of aggregate output and the comovement of sectoral output.

Consumption-good, **investment-good**, **and intermediate-input retailers** The consumptiongood retailer differs from that of the stylized model in that the elasticity of substitution of consumption across sectors is allowed to be non-unitary. More specifically, we assume that

$$C_{t} = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_{C}}} C_{s,t}^{\frac{\nu_{C}-1}{\nu_{C}}}\right]^{\frac{\nu_{C}}{\nu_{C}-1}},\tag{19}$$

where ν_C is the elasticity of substitution of consumption across sectors, and $\omega_{C,s}$ denotes the weight of good s in the consumption bundle, such that $\sum_{s=1}^{S} \omega_{C,s} = 1$. Analogously, we consider a representative investment-good retailer, which assembles the aggregate investment good, I_t , that is then sold to the household. In this case, the elasticity of substitution is ν_I , while the sectoral weights are denoted by $\omega_{I,s}$.

Finally, a representative intermediate-input retailer assembles the goods supplied by the wholesalers of all sectors into a bundle of intermediate inputs destined exclusively to the producers of a specific sector. The representative intermediate-input retailer that sells exclusively to sector s produces the bundle $H_{s,t}$ using the CES technology

$$H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x}^{\frac{1}{\nu_{H}}} H_{s,x,t}^{\frac{\nu_{H}-1}{\nu_{H}}}\right]^{\frac{\nu_{H}}{\nu_{H}-1}},$$
(20)

where $H_{s,x,t}$ is the quantity of goods purchased from the wholesaler of sector x, ν_H is the elasticity of substitution of intermediate inputs across sectors, and $\omega_{H,s,x}$ is the weight of the intermediate inputs produced by sector x in the total amount of intermediate inputs used by firms in sector s, such that $\sum_{x=1}^{S} \omega_{H,s,x} = 1$.

Government The government consists of a monetary and a fiscal authority. The monetary authority sets the nominal interest rate following a standard Taylor rule that responds to aggregate inflation and the aggregate output gap. The fiscal authority purchases goods from each sector. The amount of government spending in sector s is governed by the following auto-regressive process:

$$\log G_{s,t} = (1 - \rho) \log G_s^* + \rho \log G_{s,t-1} + v_{s,t}, \tag{21}$$

where $\rho \in (0, 1)$ measures the persistence of the process. Sectoral government spending changes over time following the realizations of the unique source of uncertainty in the model: sectoral government spending shocks, $v_{s,t}$, which are zero-mean innovations. As in the stylized model, government purchases are financed through lump-sum taxes paid by the households.

2.2.2 Calibration

Parameter values We calibrate the model to the U.S. economy, assuming that it consists of S = 57 sectors, which roughly correspond to the 3-digit level of the NAICS code list.²³ Throughout the

 $^{^{23}}$ The list of sectors is reported in Appendix C.

analysis, we assume that one period in the model corresponds to a quarter. In what follows, we describe the calibration of the parameters that govern sectoral heterogeneity and interaction in the model, as well as those characterizing the exogenous process of government spending. Appendix C describes the calibration of the remaining parameters.

We set the elasticity of substitution of consumption across sectors to $\nu_C = 2$, in line with the estimates reported by Hobijn and Nechio (2019) based on the 2-digit and 3-digit levels of disaggregation of the expenditure categories included in the calculation of the Harmonized Index of Consumer Prices. The same value is assigned to the elasticity of substitution of investment across industries, i.e., $\nu_I = 2$. We set the elasticity of substitution of intermediate inputs across sectors following the estimates of Barrot and Sauvagnat (2016), Atalay (2017), and Boehm, Flaaen and Pandalai-Nayar (2019), who find a strong degree of complementarity across industries, and do not reject the hypothesis that the aggregator of intermediate inputs is a Leontief function. Accordingly, we set $\nu_H = 0.1$.

We calibrate the sectoral weights $\omega_{C,s}$, $\omega_{I,s}$, and $\omega_{H,s,x}$ using the information of the Input-Output Tables of the U.S. economy provided by the Bureau of Economic Analysis. The consumption weights, $\omega_{C,s}$, target the average contribution of each sector to personal consumption expenditures over the period 1997-2015. Analogously, the investment weights, $\omega_{I,s}$, and the intermediate-input weights, $\omega_{H,s,x}$, target the average contribution of each sector to nonresidential private fixed investment in structures and in equipment, and the average use of intermediate inputs from sector x in the production of sector s, respectively.²⁴ The joint calibration of $\omega_{C,s}$, $\omega_{I,s}$, and $\omega_{H,s,x}$ allows the model to match the sectoral shares in private final demand.

To set the factor intensities, $\alpha_{N,s}$ and $\alpha_{H,s}$, we use information from the Input-Output Tables on value added, labor compensation, and use of intermediate inputs. More specifically, we posit that the gross output of each sector equals the sum of the compensation of employees, the gross operating surplus, and the cost of intermediate inputs.²⁵ Since we consider a constant-return-to-scale Cobb-Douglas production function for gross output, we can compute $\alpha_{H,s}$ as the sectoral share of intermediate inputs in gross output (net of the share accrued to the markup). Analogously, we set $\alpha_{N,s}$ as the sectoral share of the compensation of employees in value added.

We calibrate the sectoral elasticities of substitution across varieties based on the markup estimates obtained by Loecker, Eeckhout and Unger (2020) using firm-level data for the U.S. economy. To assign values to the sectoral Calvo probabilities, ϕ_s , we match our sectors with the items/industries analyzed by Nakamura and Steinsson (2008) and Bouakez, Rachedi and Santoro (2023), and rely on their estimates of the sectoral durations of price spells to back out the values of ϕ_s . Following Horvath (2000), we set the parameter governing the elasticity of substitution of labor across sectors to $\nu_N = 1.^{26}$ Analogously, we set $\nu_K = 1$. We calibrate the weights $\omega_{N,s}$ and $\omega_{K,s}$ such that the model features identical wages and rental rates of capital across sectors in the steady state. To do so, we set $\omega_{N,s} = \frac{N_s^*}{N^*}$ and $\omega_{K,s} = \frac{K_s^*}{K^*}$.

²⁴The calibration of the sectoral weights is conditional on the values of the elasticities ν_C , ν_I , and ν_H .

 $^{^{25}\}mathrm{We}$ leave out taxes and subsidies from the computation of gross output.

²⁶Our calibration choice for ν_N and ν_K implies a larger extent of labor and capital reallocation across sectors than in the case where these inputs are either firm- or sector-specific (e.g., Matheron, 2006; Altig et al., 2011; Carvalho and Nechio, 2016).

We normalize total government spending such that it sums up to 20 percent of aggregate value added in the steady state. Finally, we set the autoregressive parameter of the sectoral processes of government spending to $\rho = 0.90$, and calibrate the steady-state sectoral government purchases, G_s^* , using information from the Input-Output Tables on the average contribution of each industry to general government purchases.

Appendix C.2 reports an exercise aimed at validating the goodness of our calibration strategy. Specifically, we compare a non-targeted moment of U.S. sectoral data, namely the partial-equilibrium cross-industry elasticity of output to government purchases, with an analogous statistic estimated based on simulated data from our calibrated model.

2.2.3 Counterfactual experiments

We use the calibrated model to determine the way in which a government spending shock in a given sector affects aggregate value added, depending on that sector's characteristics. To parallel the analysis based on the stylized model, we study the role of the sectoral contribution to final demand, measured by the consumption and investment weights, $\omega_{C,s}$ and $\omega_{I,s}$, respectively, and size distortion, which depends on the steady-state markup, ϑ_s , labor intensity, $\alpha_{N,s}$, and position in the network.²⁷ In addition, we examine the role of the sectoral degree of price stickiness, ϕ_s . While several studies have emphasized the importance of sectoral heterogeneity in price rigidity in amplifying the macroeconomic effects of aggregate demand shocks (e.g., Carvalho, 2006; Nakamura and Steinsson, 2010; Bouakez, Cardia and Ruge-Murcia, 2014; Bouakez, Rachedi and Santoro, 2023), little is known about how the aggregate implications of a demand shock in a given sector depend on its pricing frictions.

To take into consideration the dynamic nature of the response of aggregate value added to spending shocks, we henceforth follow the standard practice of measuring this response as a multiplier, which we compute as the present-value change in aggregate output resulting from a dollar increase in government purchases from a given sector (e.g., Uhlig, 2010). That is, for a spending shock originating in sector s, the aggregate value-added multiplier is given by²⁸

$$\mathcal{M}_{G_s} = \frac{\sum_{j=0}^{\infty} \beta^j \left(Y_{t+j} - Y^* \right)}{\sum_{j=0}^{\infty} \beta^j \left(Q_{s,t} G_{s,t+j} - Q_s^* G_s^* \right)}, \qquad s = 1, \dots, S.$$
(22)

We perform counterfactual simulations by computing the spending multiplier in a sequence of model versions that allow for one dimension of sectoral heterogeneity at a time, while imposing symmetry in all the remaining sectoral attributes. The scatter plots relating the aggregate multiplier to the sectoral

$$\mathbf{c} = \frac{\alpha_H}{S} \left(\mathbf{I} - \alpha_H \mathbf{W}' \right)^{-1} \mathbf{1}$$

²⁷To rank the sectors' positions in the production network, we construct the Katz-Bonacich measure of centrality for each industry:

where α_H is the average gross-output intensity of intermediate inputs, S is the number of sectors, \mathbf{I} is a diagonal matrix, $\mathbf{W} = \{\omega_{H,s,x}\}_{s,x=1}^{S}$ is the Input-Output matrix of economy, and $\mathbf{1}$ is a vector of ones. According to this measure, more central industries are those located upstream in the network, supplying most of the intermediate inputs to the other industries. Instead, sectors with low levels of centrality are mainly users of intermediate inputs and are therefore located downstream.

²⁸Thus, the effects quantified in this section correspond to those of an additional dollar spent by the government, whereas those discussed in the analytical results are those of an additional unit of good purchased by the government.

characteristics are depicted in Figure 1. The figure shows the results based on the calibrated multisector model (to which we refer as the baseline model) as well those predicted by a counterfactual economy with perfect labor and capital mobility.

The baseline model predicts that the aggregate value-added multiplier is larger when spending shocks originate in sectors with a relatively small contribution to final demand, low markup, high labor intensity, and in those located downstream in the production network. These findings corroborate the analytical results discussed in Section 2. Quantitatively, the multiplier varies from roughly 0.52 to 0.62 as a function of the consumption share, from 0.45 to 0.66 as a function of the investment share, from 0.41 to 0.65 as a function of the markup, from 0.51 to 0.62 as a function of labor intensity, and from 0.45 to 0.59 as a function of centrality. Instead, heterogeneity in price rigidity generates limited variation in the aggregate multiplier: while the sectoral Calvo probabilities span almost the entire range of possible values, the multiplier varies only from 0.55 to 0.61, and is flat over most of the spectrum of these probabilities.^{29,30}

Next, consider the model with perfectly mobile labor and capital. In this environment, the shocked sector's contribution to consumption, investment, and government spending, as well as its degree of centrality, are irrelevant for the aggregate spending multiplier, just as predicted by the stylized model. Moreover, the multiplier, albeit not constant, is barely sensitive to the sectoral degree of price rigidity. On the other hand, the multiplier is larger when spending originates in sectors with lower markups and higher labor intensities. To understand why the latter prediction deviates from that of the stylized model results, notice that the baseline model relaxes the assumption of an infinite Frisch elasticity of labor supply (i.e., $\eta = 0$), which in turn implies that relative-price adjustment is no longer a necessary condition for the sectoral origin to matter. When $\eta > 0$, the response of aggregate value added also depends on the resource-constraint effect, as the aggregate real wage depends on aggregate labor demand. The fact that the multiplier still falls with the markup and rises with labor intensity even when relative prices are unresponsive (as a result of perfect labor and capital mobility) indicates that the resource-constraint effect strengthens the role of these two characteristics in accounting for the dispersion in the aggregate output effects of sectoral spending shocks.

2.3 Testing the theoretical predictions

In this section, we provide suggestive evidence supporting our theoretical predictions that trace the aggregate effects of government spending shocks to the structural characteristics of the sectors in which those shocks originate. These characteristics are the contribution to consumption and investment (measured by ω_C and ω_I , respectively), steady-state markup, ϑ , labor intensity, α_N , position in the network (measured by centrality, c), and degree of price rigidity (measure by the Calvo probability,

²⁹In a companion paper (Bouakez, Rachedi and Santoro, 2023), we show that heterogeneity in the sectoral degree of price rigidity raises the size of the aggregate multiplier relative to an economy with a common average duration of prices across industries. Hence, this dimension of sectoral heterogeneity acts mainly as a level shifter, but has little bearing on the heterogeneity in the spending multipliers associated with sector-specific shocks.

³⁰These findings are quantitatively similar to those obtained in the case of immobile labor and capital ($\nu_N = \nu_K = 0$), as reported in Appendix D, thus confirming that our results are not driven by changes in the economy-wide allocative efficiency induced by the reallocation of production factors across sectors outside the steady state.



Figure 1: Sectoral Characteristics and the Aggregate Value-Added Multiplier.

Notes: The figure reports the aggregate value-added multiplier in counterfactual economies in which we allow for one dimension of sectoral heterogeneity at a time, while imposing symmetry in all the remaining sectoral attributes. We consider both the baseline case of imperfect mobility of labor and capital (black dots), as well as the extreme case of perfect mobility (red squares). Each dot/square represents one of the 57 sectors of the economy.

 ϕ). Given that endogeneity issues hamper a direct approach that would consist in regressing changes in national aggregate output on measures of sectoral government purchases, we opt for an alternative that exploits variation in public procurement across U.S. states. More specifically, suppose that, theoretically, the aggregate output multiplier is increasing in a given sectoral characteristic C. The argument underlying our empirical approach is that the relative (or local) multiplier of government purchases in a given state *i* should be larger the more those purchases are concentrated in sectors with a relatively high C.

To implement this strategy, we retrieve comprehensive data on U.S. federal defense spending from www.usaspending.gov to construct a dataset comprising information at both the sectoral level (as classified by the Bureau of Economic Analysis) and at the state level. For each year, sectoral data are then aggregated at the state level, spanning the 2001-2021 time window. The resulting panel is used to estimate, for each of the six sectoral characteristics listed above, the following regression:

$$\frac{Y_{i,t} - Y_{i,t-2}}{Y_{i,t-2}} = \beta_1 \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \beta_2 \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(\mathcal{C}_{i,t} - \bar{\mathcal{C}}_{i,t}\right) + \beta_3 \mathcal{C}_{i,t} + \alpha_i + \delta_t + \epsilon_{i,t}, \tag{23}$$

where $Y_{i,t}$ and $G_{i,t}$ are, respectively, real GDP per-capita and real federal defense spending per-capita in state *i* and year *t*, $C_{i,t}$ denotes the weighted-average sectoral characteristic C in state *i* and year t,³¹ and $\bar{C}_{i,t} = \frac{1}{\bar{L}} \frac{1}{\bar{L}} \sum_{i=1}^{\bar{L}} \sum_{t=1}^{\bar{L}} C_{i,t}$ is the mean of the weighted-average sectoral characteristic C.³² The regression also includes state fixed effects, α_i , and year fixed effects, δ_t .³³ To test our predictions, we interact our measure of the government spending shock with the demeaned value of a the characteristic of interest. Thus, the estimate of β_1 captures the local spending multiplier — defined as the dollar response in the GDP of state *i* following a dollar increase in federally-financed defense spending in that state — and the value of β_2 informs us on how the local multiplier varies with the different dimensions of sectoral heterogeneity. The predicted sign of β_2 is therefore negative for ω_C , ω_I , ϑ , and *c*, but positive for α_N and ϕ .

Because state-level defense purchases are likely to depend on the economic conditions of the recipient state, we identify state-level shocks to public spending by assuming that the allocation of aggregate defense spending across states remains constant over time. More specifically, we use a shift-share approach as in Nakamura and Steinsson (2014) and Auerbach, Gorodnichenko and Murphy (2020), and instrument $\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$ with the product of the 2-year growth rate in real federal defense spending at the national level, $\frac{G_t-G_{t-2}}{G_{t-2}}$, with the average ratio of nominal federal defense spending over nominal GDP, $\frac{G_i}{Y_i}$.³⁴ We then multiply this Bartik variable by the demeaned characteristic of interest to instrument the interaction term.

Table 1 reports the estimation results for the coefficients β_1 and β_2 . Baseline refers to the esti-

³¹For any given sectoral characteristic C, $C_{i,t} = \sum_{s=1}^{S} \gamma_s \frac{\mathcal{G}_{s,i,t}}{\mathcal{G}_{i,t}}$, where $\frac{\mathcal{G}_{s,i,t}}{\mathcal{G}_{i,t}}$ is the ratio between nominal federal defense spending in sector s, in state i and year t, over total nominal federal defense spending in state i and year t.

 $^{^{32}}$ We denote by I the number of states in our sample, and with T the length of the time-series dimension of the panel.

³³Time fixed effects absorb the national general-equilibrium effects of state-level spending shocks. Thus, Equation (23) would not allow one to estimate the model-based aggregate multiplier, even if the model economy featured multiple states. However, estimates of β_2 are still informative about the direction in which the sectoral composition of spending in a given state affects its output relative to national output.

 $^{^{34}}$ We compute this ratio by averaging over the first 4 years of our sample, from 2001 to 2004.

		Characteristics					
		Contribution to consumption	Contribution to investment	Markup	Labor intensity	Centrality	Price rigidity
	Shock	1.626	1.378	1.395	1.432	1.5183	1.430
Baseline		(0.004)	(0.020)	(0.014)	(0.013)	(0.009)	0.012
	Interaction term	-7.116	-2.275	-0.199	0.198	-3.410	0.0120
		(0.030)	(0.125)	(0.025)	(0.021)	(0.010)	(0.018)
Tax control	Shock	1.333	1.229	1.129	1.193	1.242	1.190
		(0.021)	(0.041)	(0.056)	(0.044)	(0.035)	(0.043)
	Interaction term	-6.643	-2.769	-0.191	0.200	-3.249	0.012
		(0.028)	(0.077)	(0.031)	(0.022)	(0.014)	(0.020)
Passling (0001 0010)	Shock	1.594	1.388	1.413	1.526	1.543	1.499
Baseline (2001-2019)		(0.004)	(0.011)	(0.014)	(0.013)	(0.009)	(0.014)
	Interaction term	-6.410	-1.832	-0.197	0.196	-3.237	0.010
		(0.044)	(0.106)	(0.035)	(0.034)	(0.017)	(0.038)
Tax control (2001-2019)	Shock	1.252	1.150	1.065	1.199	1.190	1.172
		(0.024)	(0.038)	(0.072)	(0.056)	(0.045)	(0.060)
	Interaction term	-5.918	-2.176	-0.180	0.190	-2.964	0.010
		(0.055)	(0.072)	(0.051)	(0.043)	(0.028)	(0.050)

Table 1: Local Spending Multiplier and Sectoral Characteristics.

Notes: The table reports the estimates of coefficients β_1 and β_3 in Equation (23), as well as the corresponding p-values (between parentheses). Estimates are based on a state-level panel, measured at an annual frequency, from 2001 to 2021. Standard errors clustered at the state level. *Baseline* refers to the estimation considering the entire sample, while *Tax controls* refers to the case in which we control for the contemporaneous change in the ratio of personal federal taxes over personal income. The additional label 2001-2019 refers to the estimations that exclude the years 2020 and 2021.

mation considering the entire sample, while *Tax controls* refers to the case in which we control for the contemporaneous change in the ratio of personal federal taxes over personal income. In both cases, we also consider a restricted sample period that ends before the start of the COVID-19 episode (i.e., that excludes the years 2020 and 2021), and refer to this configuration using the additional 2001-2019 label.

In the baseline regressions, all the estimates of β_2 (associated with the interaction term) have the correct sign and, with one exception, are significant at conventional statistical levels. Numerically, the local multiplier displays very little sensitivity to heterogeneity in the price rigidity of the goods and services bought by the Department of Defense. These results remain remarkably robust when we control for taxes and/or exclude the post-2019 period. In fact, in the regressions that include the tax control, all the estimates of β_2 are statistically significant, indicating that the local multiplier tends to be larger when government purchases from a given state are more skewed towards sectors that contribute relatively less to private final demand, have lower markups, are labor intensive, and are located downstream in the supply chain.

3 Quantitative Analysis

Having shown that the industry primitives shape the output effects of sectoral spending shocks in a way that is consistent with the data, we now leverage the fully fledged structure of the calibrated model presented in Section 2.2 to provide quantitative insights into two corollaries of our findings. First, we

measure the dispersion in the aggregate multiplier associated with U.S. sectoral government purchases. Second, we evaluate the extent to which differences in the sectoral composition of purchases across U.S. government levels (i.e., general, federal, and state & local) translate into significant differences in the spending multiplier.

3.1 Dispersion of the aggregate spending multiplier

So far, we have studied the aggregate effects of sectoral spending shocks in environments that only allow for one source of sectoral heterogeneity at a time. In what follows, we use the calibrated model to determine whether the heterogeneity across U.S. industries implies substantial dispersion in the size of the aggregate spending multiplier associated with sectoral government purchases. Such an exercise is not only useful to quantify the extent to which the aggregate effects of government purchases depend on their sectoral origin, but also to identify the industries associated with a large "bang for the buck". In that regard, our findings can inform policymakers about the industries they should target to maximize the aggregate spending multiplier.

Figure 2 presents a bar plot of the aggregate multipliers associated with the different sectoral spending shocks, taken in isolation. The multiplier ranges from 0.23 when government spending originates in machinery manufacturing to 1.13 when it originates in legal services. In general, the multiplier is lower when the government buys from upstream industries with a relatively large contribution to investment and low labor intensity, like manufacturing. On the other hand, it is larger when the government raises its demand for goods produced by downstream labor-intensive industries that contribute little to investment, such as retail trade, and educational and health-care services.



Figure 2: Aggregate Output Multiplier of Sectoral Government Spending Shocks.

Notes: The figure plots the aggregate value-added multiplier associated with each sectoral government spending shock, obtained from the fully heterogeneous economy.

Figures 3 and 4 report the bar plots associated with the aggregate consumption and investment



Figure 3: Aggregate Consumption Multiplier of Sectoral Government Spending Shocks.

Notes: The figure plots the aggregate consumption multiplier associated with each sectoral government spending shock, obtained from the fully heterogeneous economy.





Notes: The figure plots the aggregate investment multiplier associated with each sectoral government spending shock, obtained from the fully heterogeneous economy.

multipliers. While both multipliers are correlated with that of aggregate output, the consumption multiplier exhibits much larger dispersion — as compared with the investment multiplier — with values ranging from -0.36 for purchases from motor vehicles to 0.44 for purchases from legal services. Interestingly, our assumption of complementarity between aggregate consumption and total govern-

ment purchases does not affect the dispersion of the aggregate multipliers, as it only shifts upward the response of aggregate variables to all sectoral shocks (see Figures E.2–E.3 in Appendix E).³⁵ In doing so, however, it generates heterogeneity in the sign of the response of aggregate consumption, implying a negative consumption multiplier for 23 of the 57 sectoral spending shocks. Instead, the aggregate investment multiplier is uniformly negative across all sectoral shocks, and ranges from -0.27for spending on educational services to -0.44 for purchases from the construction sector.

These findings underscore the importance of conditioning on the nature of government purchases when estimating their macroeconomic effects. Depending on the sectoral origin of the spending shock, the output multiplier can be small or large, and the response of aggregate consumption can be either positive or negative. From this perspective, our approach has the potential to reconcile divergent views about the effectiveness of spending-based fiscal policy. The next section sheds further light on this issue.

3.2 The sectoral composition of government purchases across government levels

The heterogeneity in the size of the aggregate spending multiplier across sectoral spending shocks implies that changes in the sectoral allocation of government spending translate into changes in the multiplier. To measure the extent to which the sectoral composition of government spending matters, we evaluate the aggregate output and consumption multipliers associated with public purchases at the different layers of the U.S. government — general, federal (defense and non-defense), and state & local. For each of these levels, the shares of government spending pertaining to the 57 sectors are computed based on the entries of the Input-Output Tables of the U.S. Bureau of Economic Analysis, averaged over the period 1997–2015. In evaluating the aggregate multipliers associated with the different government levels, we consider otherwise identical versions of the quantitative model that differ only in the spending shares assigned to the different sectors outside the steady state.³⁶ In this way, we can isolate differences in the multipliers that are to be attributed to differences in the sectoral differences in the spending shock (i.e., spending in excess of its steady-state level).^{37,38}

Figure 5 reports the aggregate output and consumption multipliers associated with the different levels of the U.S. government. An additional dollar spent by the general government raises aggregate value added by 72 cents. This value, however, masks substantial heterogeneity in the output effects of spending across government levels. The aggregate output multiplier associated with spending by the

³⁵The correlation between the aggregate output multipliers of sectoral spending implied by the models with and without complementarity is 0.98.

³⁶All the deep parameters are calibrated identically across the model versions we consider, which therefore share the same steady state. We compute the spending multipliers associated with different linear combinations of sectoral spending shocks, where the weights replicate the average sectoral composition of purchases at each of the U.S. government levels.

³⁷It should be emphasized that our analysis is strictly positive in nature, and does not imply ranking the sectoral allocations of government spending at the different levels of the U.S. government in terms of their desirability from a welfare perspective.

³⁸In Appendix F, we report the results from a similarly devised exercise, in which we compare the aggregate output and consumption multipliers implied by the quantitative model based on the sectoral composition of national government spending in 28 OECD countries. We show that the spending multiplier of the U.S. economy could be as low as 0.54 if additional spending by the U.S. government had the same sectoral composition as that of the general government of Slovakia, and as high as 0.95 if it reflected the sectoral composition of purchases of the United Kingdom's general government.



Figure 5: The Aggregate Spending Multiplier across Government Levels.

Notes: The figure reports the aggregate value-added multipliers across the different levels of the U.S. government. The multipliers are computed from otherwise identical economies that only differ the sectoral composition of the government spending shock, which is calibrated based on the Input-Output Tables of the U.S. Bureau of Economic Analysis.

state & local government amounts to 0.82, and is 46 percent larger than that associated with spending by the federal government, which equals 0.58. There is large variation in the output multipliers even within federal spending: the one associated with non-defense spending amounts to 0.70, and is almost 50 percent larger than that associated with defense spending, which equals 0.47.

A similar pattern holds for the aggregate consumption multiplier, which is close to 0 when measured at the general-government level, but negative (-0.10) for federal spending and positive (0.13) for state & local spending. A large disparity in the consumption multiplier is also observed when federal purchases are split into defense and non-defense spending. The multiplier amounts to -0.17 for the former and 0 for the latter.

To understand the large variation in the aggregate output multiplier across the different levels of the U.S. government, Figure 6 reports the sectoral composition of state & local, federal non-defense, and defense spending. For expositional simplicity, we look at eight macro industries: primary (including agriculture, mining, utilities, and construction), manufacturing, trade, transportation, information, professional services, education and health-care services, and other services (including arts, recreation services, and food and hotel services). Defense spending is largely concentrated in manufacturing industries, for which we found lower multipliers. Instead, both federal non-defense and state & local spending are mostly oriented towards services, which have relatively large multipliers.

These results imply that the spending multiplier crucially depends on the sectoral composition of government purchases. This observation has consequential implications for empirical work on the effects of fiscal policy. First, it may provide a rationale for the wide range of estimates of the spending multiplier reported in the literature: Studies that rely on federal defense spending tend to report small



Figure 6: The Sectoral Composition of Government Spending in the United States.

Notes: The figure reports the sectoral shares of government spending at the different levels of the U.S. government across eight macro-industries. Primary refers to farms, forestry, mining, utilities, and construction. Other services include arts, recreation industries, accommodation services, and food services. The shares are computed based on the Input-Output Tables of the U.S. Bureau of Economic Analysis.

output multipliers and a crowding-out of consumption (e.g., Barro and Redlick, 2011; Ramey, 2011); instead, measuring spending at the general-government level typically leads to large output multipliers and a crowding-in of consumption (e.g., Blanchard and Perotti, 2002; Auerbach and Gorodnichenko, 2012).³⁹ Second, our analysis poses yet another challenge for the identification of government spending shocks based on times-series data, which should not only exploit exogenous shifts in total government spending, but also control for time variation in the sectoral allocation of public expenditure.

4 Conclusion

This paper has explored the role of sectoral characteristics and network linkages in shaping the aggregate effects of sector-specific government spending shocks. Based on a tractable multi-sector model, we have derived a formula that characterizes the response of aggregate output to an increase in government purchases from a given sector. Our analytical results indicate that sectoral spending shocks tend to have larger aggregate output effects when they originate in sectors that have a relatively small contribution to private final demand, low markup, high labor intensity, and in those located downstream in the production network. These insights carry over to a quantitative multi-sector model that incorporates several realistic dimensions of sectoral heterogeneity and input-output interactions, which we calibrate to the U.S. economy. The model implies substantial heterogeneity in the aggregate

³⁹While our analysis controls for the financing scheme and the stance of monetary policy when comparing the aggregate multipliers associated with the different sectoral shocks, these considerations can play a role in explaining the difference in the empirical estimates reported in the literature.

output and consumption multipliers of sectoral spending shocks. It also reveals sizable differences in the aggregate multipliers across the different layers of the U.S. government. Together, these findings have important implications regarding the sectoral composition of spending-based stimulus plans that aim at generating sizable "bangs for the buck".

As in the vast majority of papers studying the spending multiplier, we have focused on a specific component of government consumption: purchases of goods and services from the private sector. While a branch of the literature studies the aggregate effects of government employment/wages (e.g., Finn, 1998; Pappa, 2009; Chang et al., 2021), little is known, however, about the government aggregate production function that regulates the interaction between public employment and public purchases. In the spirit of Baqaee and Farhi (2018), our bottom-up approach could in principle help micro-found such a construct. Unfortunately, data on public employment are not available at the same level of sectoral disaggregation as for government purchase, precluding researchers from pursuing this line of inquiry.⁴⁰

Finally, while this paper proposes a novel perspective to think about the transmission of government spending, the analysis has remained positive in nature. The marked heterogeneity in the aggregate effects of sectoral public spending, however, suggests that, from a normative standpoint, an optimizing fiscal authority needs to determine not only the optimal level of government spending, but also its composition. We leave this issue for future research.

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⁴⁰This data limitation might explain why only a handful of papers allow for the interaction between public purchases and public employment through an aggregate production function of the government (e.g., Moro and Rachedi, 2022).

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Appendix

A Stylized Model

This appendix solves the stylized model employed to derive the analytical results discussed in Section 2, and reports the proofs to the propositions contained therein.

A.1 Non-linear economy

This subsection reports the necessary set of equations to solve the stylized model. From households' optimal allocation between consumption and labor hours we obtain

$$\theta = C^{-1}W,\tag{A.1}$$

where $W = \mathcal{W}/P$ is the aggregate real wage and $P = \omega_{C,d}^{-\omega_{C,d}} \omega_{C,u}^{-\omega_{C,u}} P_d^{\omega_{C,u}} P_u^{\omega_{C,u}}$ is the aggregate price level. The optimal consumption demand for the goods produced by the two sectors are given by

$$C_u = \omega_{C,u} \frac{C}{Q_u}, \tag{A.2}$$

$$C_d = \omega_{C,d} \frac{C}{Q_d}, \tag{A.3}$$

where $Q_s = \frac{P_s}{P}$ for s = u, d.

Households' supply of labor to the two sectors is given by

$$N_u = \omega_{N,u} \left(\frac{W_u}{W}\right)^{\nu_N} N, \tag{A.4}$$

$$N_d = \omega_{N,d} \left(\frac{W_d}{W}\right)^{\nu_N} N, \tag{A.5}$$

where $W_s = W_s/P$ is the real wage in sector s (s = u, d). The aggregate real wage satisfies

$$W = \left[\omega_{N,d} W_d^{1+\nu_N} + \omega_{N,u} W_u^{1+\nu_N}\right]^{\frac{1}{1+\nu_N}}.$$
 (A.6)

Recall that the sectoral production technologies read as

$$Z_u = N_u^{1-\alpha_{H,u}} H_{u,u}^{\alpha_{H,u}}, \tag{A.7}$$

$$Z_d = N_d^{1-\alpha_{H,d}} H_{d,u}^{\alpha_{H,d}}.$$
 (A.8)

Thus, producers' cost-minimization returns the following first-order conditions:

$$W_u = (1 - \alpha_{H,u}) \frac{MC_u Z_u}{N_u}, \qquad (A.9)$$

$$Q_u = \alpha_{H,u} \frac{MC_u Z_u}{H_{u,u}}, \tag{A.10}$$

$$W_d = (1 - \alpha_{H,d}) M C_d \frac{Z_d}{N_d},$$
 (A.11)

$$Q_u = \alpha_{H,d} \frac{MC_d Z_d}{H_{d,u}},\tag{A.12}$$

where MC_s is the real marginal cost of production in sector s, which satisfies $Q_s = \vartheta_s MC_s$, where $\vartheta_s \equiv \frac{\epsilon_s}{\epsilon_s - 1}$ denotes the sectoral markup (s = u, d). Finally, the model is closed by the sectoral resource constraints

$$Z_u = C_u + G_u + H_{u,u} + H_{d,u}, (A.13)$$

$$Z_d = C_d + G_d. \tag{A.14}$$

A.2 Steady state

This sub-section describes the steady state. Steady-state variables are denoted by an asterisk. We start by rearranging the production function of sector u as

$$1 = \left(\frac{N_u^*}{Z_u^*}\right)^{1-\alpha_{H,u}} \left(\frac{H_{u,u}^*}{Z_u^*}\right)^{\alpha_{H,u}}.$$
(A.15)

From (A.9) and (A.10), we have

$$\frac{N_u^*}{Z_u^*} = (1 - \alpha_{H,u}) \frac{MC_u^*}{W_u^*} = \vartheta_u^{-1} (1 - \alpha_{H,u}) \left(\frac{W_u^*}{Q_u^*}\right) \quad \text{and} \quad \frac{H_{u,u}^*}{Z_u^*} = \alpha_{H,u} \frac{MC_u^*}{Q_u^*} = \vartheta_u^{-1} \alpha_{H,u}, \quad (A.16)$$

which can combined with (A.15) to obtain

$$\left(\frac{W_u^*}{Q_u^*}\right)^{1-\alpha_{H,u}} = \vartheta_u^{-1} \left(1-\alpha_{H,u}\right)^{1-\alpha_{H,u}} \alpha_{H,u}^{\alpha_{H,u}}.$$
(A.17)

Analogously, the following result holds for sector d:

$$\left(\frac{W_d^*}{Q_d^*}\right)^{1-\alpha_{H,d}} = \vartheta_d^{-1} \left(1-\alpha_{H,d}\right)^{1-\alpha_{H,d}} \alpha_{H,d}^{\alpha_{H,d}} \left(\frac{Q_d^*}{Q_u^*}\right)^{\alpha_{H,d}}.$$
(A.18)

Under our calibration $\omega_{N,s} = \frac{N_s^*}{N^*}$ for s = u, d, (A.4) and (A.5) imply that $W_u^* = W_d^* = W^*$. Therefore, (A.17) and (A.18) lead to

$$\frac{Q_{d}^{*}}{Q_{u}^{*}} = \frac{\left[\vartheta_{u}^{-1} \left(1 - \alpha_{H,u}\right)^{1 - \alpha_{H,u}} \alpha_{H,u}^{\alpha_{H,u}}\right]^{\frac{1 - \alpha_{H,u}}{1 - \alpha_{H,u}}}}{\vartheta_{d}^{-1} \left(1 - \alpha_{H,d}\right)^{1 - \alpha_{H,d}} \alpha_{H,d}^{\alpha_{H,d}}}$$

Let $\omega_{G,s} \equiv \frac{Q_s^* G_s^*}{Q_u^* G_u^* + Q_d^* G_d^*}$ denote the steady-state share of government spending allocated to sector s (s = u, d), and $\gamma \equiv \frac{Q_u^* G_u^* + Q_d^* G_d^*}{Y^*}$ denote the share of total government spending in total value added, Y^* , it follows that

$$Q_u^* G_u^* = \gamma \omega_{G,u} Y^*, \tag{A.19}$$

$$Q_d^* G_d^* = \gamma \omega_{G,d} Y^*, \tag{A.20}$$

and

$$Q_d^* C_d^* = (1 - \gamma) \,\omega_{C,d} Y^*, \tag{A.21}$$

$$Q_u^* C_u^* = (1 - \gamma) \,\omega_{C,u} Y^*.$$
(A.22)

Noting that $Y_d^* = Q_d^* Z_d^* - Q_u^* H_{d,u}^* = (1 - \vartheta_d \alpha_H) Q_d^* Z_d^* = (1 - \vartheta_d \alpha_H) (Q_d^* C_d^* + Q_d^* G_d^*)$ and using the fact that $\frac{Y_u^*}{Y^*} + \frac{Y_d^*}{Y^*} = 1$, we obtain the following expression for the sectoral value added as a fraction of aggregate value added:

$$\frac{Y_u^*}{Y^*} = 1 - (1 - \tilde{\alpha}_{H,d}) \,\mu_d, \tag{A.23}$$

$$\frac{Y_d^*}{Y^*} = (1 - \tilde{\alpha}_{H,d}) \mu_d, \tag{A.24}$$

where $\tilde{\alpha}_{H,d} \equiv \alpha_{H,d} \vartheta_d^{-1}$ and $\mu_d \equiv \frac{Q_d^* C_d^* + Q_d^* G_d^*}{Y^*} = (1 - \gamma) \omega_{C,d} + \gamma \omega_{G,d}$ is the steady-state contribution of sector d to total final demand.

Moreover, (A.9) and (A.11) imply that $\frac{N_u^*}{(1-\alpha_{H,u})\vartheta_u^{-1}Q_u^*Z_u^*} = \frac{N_d^*}{(1-\alpha_{H,d})\vartheta_d^{-1}Q_d^*Z_d^*}$. Since $Y_d^* = (1-\alpha_H\vartheta_d^{-1})Q_d^*Z_d^*$, $Y_u^* = (1-\alpha_H\vartheta_u^{-1})Q_u^*Z_u^*$, and $N^* = N_u^* + N_d^*$, it follows that

$$\varpi_{u} \equiv \frac{N_{u}^{*}}{N^{*}} = \frac{(1 - \alpha_{H,u}) (\vartheta_{d} - \alpha_{H,d}) [1 - (1 - \tilde{\alpha}_{H,d}) \mu_{d}]}{(1 - \alpha_{H,u}) (\vartheta_{d} - \alpha_{H,d}) [1 - (1 - \tilde{\alpha}_{H,d}) \mu_{d}] + (1 - \alpha_{H,d}) (\vartheta_{u} - \alpha_{H,u}) (1 - \tilde{\alpha}_{H,d}) \mu_{d}}, \quad (A.25)$$

$$\varpi_{d} \equiv \frac{N_{d}}{N^{*}} = \frac{(1 - \alpha_{H,d}) (\vartheta_{u} - \alpha_{H,u}) (1 - \alpha_{H}) \mu_{d}}{(1 - \alpha_{H,u}) (\vartheta_{d} - \alpha_{H,d}) [1 - (1 - \tilde{\alpha}_{H}) \mu_{d}] + (1 - \alpha_{H,d}) (\vartheta_{u} - \alpha_{H,u}) (1 - \tilde{\alpha}_{H,d}) \mu_{d}}.$$
 (A.26)

Finally, to determine the ratio $\frac{Q_u^* H_{d,u}^*}{Y^*}$, we combine the expression for the value added of sector u (i.e., $Y_u^* = Q_u^* Z_u^* - Q_u^* Z_u^*$)

 $Q_u^* H_{u,u}^*$) with its resource constraint, obtaining

$$\frac{Y_u^*}{Y^*} = \frac{Q_u^* C_u^* + Q_u^* G_u^*}{Y^*} + \frac{Q_u^* H_{d,u}^*}{Y^*}.$$
(A.27)

Letting $\mu_u \equiv \frac{Q_u^* C_u^* + Q_u^* G_u^*}{Y^*} = (1 - \gamma) \omega_{C,u} + \gamma \omega_{G,u}$, noting that $\mu_u + \mu_d = 1$, and using (A.23), we get

$$\frac{Q_u^* H_{d,u}^*}{Y^*} = \tilde{\alpha}_{H,d} \mu_d. \tag{A.28}$$

A.3 Log-linear economy

We solve the model by log-linearizing its equilibrium conditions around the non-stochastic steady state. The log-linearized counterparts of equations (A.1)–(A.14) are, respectively:

$$c = w, \tag{A.29}$$

$$c_u = c - q_u, \tag{A.30}$$

$$c_d = c - q_d, \tag{A.31}$$

$$q_{-} = \frac{\omega_{C,u}}{\omega_{C,u}} q_{-} \tag{A.32}$$

$$q_d = -\frac{1}{\omega_{C,d}} q_u, \tag{A.32}$$

$$n_{c_u} = \nu_N (w_u - w) + n_u \tag{A.33}$$

$$n_u = \nu_N(w_u - w) + n,$$
 (A.33)

$$n_d = \nu_N(w_d - w) + n, \tag{A.34}$$

$$w = \overline{\omega}_u w_u + \overline{\omega}_d w_d, \tag{A.35}$$

$$z = (1 - \alpha u) n + \alpha u h$$
(A.36)

$$z_{u} = (1 - \alpha_{H,u})n_{u} + \alpha_{H,u}n_{u,u},$$
(A.30)
$$z_{v} = (1 - \alpha_{H,v})n_{v} + \alpha_{H,v}n_{v},$$
(A.37)

$$z_d = (1 - \alpha_{H,d})n_d + \alpha_{H,d}n_{d,u}, \tag{A.31}$$

$$w_u = q_u + z_u - n_u, \tag{A.38}$$

$$h_{u,u} = z_u, \tag{A.39}$$

$$w_d = q_d + z_d - n_d, \tag{A.40}$$

$$h_{d,u} = z_d + q_d - q_u,$$
 (A.41)

$$z_u = \frac{\gamma \omega_{G,u}}{1 - (1 - \tilde{\alpha}_{H,d}) \,\mu_d} g_u + \frac{(1 - \gamma) \,\omega_{C,u}}{1 - (1 - \tilde{\alpha}_{H,d}) \,\mu_d} c_u + \frac{\alpha_{H,d} \mu_d}{1 - (1 - \tilde{\alpha}_{H,d}) \,\mu_d} h_{d,u},\tag{A.42}$$

$$z_{d} = \frac{\gamma \omega_{G,d}}{\mu_{d}} g_{d} + \frac{(1-\gamma) \omega_{C,d}}{\mu_{d}} c_{d}.$$
 (A.43)

Derivation of equation (14) A.4

In a log-linearized form, the aggregate resource constraint is given by

$$y = (1 - \gamma) c + \gamma \omega_{G,u} \left(g_u + q_u \right) + \gamma \omega_{G,d} \left(g_d + q_d \right).$$
(A.44)

From (A.29) and (A.35), we obtain

$$c = \varpi_u w_u + \varpi_d w_d. \tag{A.45}$$

Equations (A.36) and (A.39) imply that $z_u = h_{u,u} = n_u$. In light of (A.38), this yields

$$w_u = q_u. \tag{A.46}$$

Using (A.37) to substitute for z_d in both (A.40) and (A.41), and combining the resulting two equations, we get

$$q_d = (1 - \alpha_H) w_d + \alpha_H q_u. \tag{A.47}$$

Using (A.46) and (A.47) to substitute for w_u and w_d in (A.45) returns

$$c = \frac{\overline{\omega}_u - \alpha_{H,d}}{\overline{\omega}_u^e - \alpha_{H,d}} \mu_u q_u + \frac{\overline{\omega}_d}{\overline{\omega}_d^e} \mu_d q_d, \tag{A.48}$$

where $\varpi_u^e \equiv 1 - (1 - \alpha_{H,d}) \mu_d = \alpha_{H,d} + \mu_u (1 - \alpha_{H,d})$ and $\varpi_d^e \equiv (1 - \alpha_{H,d}) \mu_d$ are the steady-state employment shares of sectors u and d, respectively, when the economy is efficient (i.e., $\vartheta_u = \vartheta_d = 1$).

Using (A.32) and the identities $\varpi_d = 1 - \varpi_u$, $\omega_{C,d} = 1 - \omega_{C,u}$, and $\mu_d = 1 - \mu_u$, we can express (A.48) as

$$c = \left[\frac{(1 - \alpha_{H,d})(\mu_u - \omega_{C,u}) - (\varpi_u^e - \varpi_u)}{(1 - \alpha_{H,d})(1 - \omega_{C,u})}\right]q_u$$
(A.49)

$$= \left[\frac{(1 - \alpha_{H,d})(\mu_d - \omega_{C,d}) - (\varpi_d^e - \varpi_d)}{(1 - \alpha_{H,d})(1 - \omega_{C,d})}\right] q_d.$$
(A.50)

Plugging (A.49) into (A.44), using (A.32), and taking the derivative with respect to g_u , we get

$$\begin{split} \frac{dy}{dg_u} &= \gamma \omega_{G,u} + \left\{ (1-\gamma) \left[\frac{(1-\alpha_{H,d}) \left(\mu_u - \omega_{C,u} \right) + \varpi_u^e - \varpi_u}{(1-\alpha_{H,d}) \left(1 - \omega_{C,u} \right)} \right] + \frac{\mu_u - \omega_{C,u}}{1 - \omega_{C,u}} \right\} \frac{dq_u}{dg_u} \\ &= \gamma \omega_{G,u} + \left[\frac{(2-\gamma) \left(\mu_u - \omega_{C,u} \right) - (1-\gamma) \left(1 - \alpha_{H,d} \right)^{-1} \left(\varpi_u^e - \varpi_u \right)}{1 - \omega_{C,u}} \right] \frac{dq_u}{dg_u}. \end{split}$$

Analogously, substituting (A.50) into (A.44), using (A.32) and taking the derivative with respect to g_d , we obtain,

$$\frac{dy}{dg_d} = \gamma \omega_{G,d} + \left\{ (1-\gamma) \left[\frac{(1-\alpha_{H,d}) \left(\mu_d - \omega_{C,d}\right) - \left(\varpi_d^e - \varpi_d\right)}{(1-\alpha_H) \left(1-\omega_{C,d}\right)} \right] + \frac{\mu_d - \omega_{C,d}}{1-\omega_{C,d}} \right\} \frac{dq_d}{dg_d}$$

$$= \gamma \omega_{G,d} + \left[\frac{(2-\gamma) \left(\mu_d - \omega_{C,d}\right) - (1-\gamma) \left(1-\alpha_{H,d}\right)^{-1} \left(\varpi_d^e - \varpi_d\right)}{1-\omega_{C,d}} \right] \frac{dq_d}{dg_d}.$$

We can therefore write

$$\frac{dy}{dg_s} = \gamma \omega_{G,s} + \left[\frac{\psi_1 \left(\mu_s - \omega_{C,s}\right) - \psi_2 \left(\varpi_s^e - \varpi_s\right)}{1 - \omega_{C,s}}\right] \frac{dq_s}{dg_s}, \quad s = u, d, \tag{A.51}$$

where $\psi_1 \equiv 2 - \gamma$ and $\psi_2 \equiv \frac{1 - \gamma}{1 - \alpha_{H,d}}$.

A.5 Proofs

Proof of Proposition 1. Part (i): From (A.51), it is immediate to see that, if $\frac{dq_s}{dg_s} = 0$, the response of aggregate value added only depends on the direct effect of the shock, which is symmetric if $\omega_{G,s} = \frac{1}{2}$ for s = u, d. Part (ii): Under efficiency, $\varpi_s^e = \varpi_s$. If, in addition, $\mu_s = \omega_{C,s}$, we again have $\frac{dy}{dg_s} = \gamma \omega_{G,s} = \frac{\gamma}{2}$ for s = u, d.

Proof of Proposition 2. Combining (A.32) and (A.41) to obtain $h_{d,u} = z_d - \frac{1}{\omega_{C,d}}q_u$, substituting the latter into (A.37), and rearranging, we get

$$z_d = n_d - \frac{\alpha_{H,d}}{(1 - \alpha_{H,d})\,\omega_{C,d}}q_u. \tag{A.52}$$

Inserting this expression into (A.43) and using (A.29), (A.31), (A.32), and (A.49) we obtain

$$n_{d} = \frac{\gamma \omega_{G,d}}{\mu_{d}} g_{d} + \frac{1-\gamma}{\mu_{d}} \left[1 - \frac{\varpi_{d}}{1-\alpha_{H,d}} + \frac{\alpha_{H,d}\mu_{d}}{(1-\gamma)\left(1-\alpha_{H,d}\right)\omega_{C,d}} \right] q_{u}$$

Combining (A.32), (A.41), and (A.52) yields

$$h_{d,u} = n_d - \frac{1}{(1 - \alpha_{H,d})\,\omega_{C,d}}q_u.$$

Substituting this expression into (A.42) and using (A.29), (A.30), and (A.49), we get

$$n_{u} = \frac{\gamma \omega_{G,u}}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_{d}} g_{u} + \frac{\gamma \tilde{\alpha}_{H,d} \omega_{G,d}}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_{d}} g_{d} + \frac{\tilde{\alpha}_{H,d} \left(1 - \gamma\right)}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_{d}} \left[1 - \frac{\varpi_{d}}{1 - \alpha_{H,d}} + \frac{\alpha_{H,d} \mu_{d}}{(1 - \gamma) \left(1 - \alpha_{H,d}\right) \omega_{C,d}} - \frac{\mu_{d} \tilde{\alpha}_{H,d} + (1 - \gamma) \varpi_{d} \omega_{C,u}}{(1 - \gamma) \left(1 - \alpha_{H,d}\right) \tilde{\alpha}_{H,d} \omega_{C,d}} \right] q_{u}$$

Using (A.46) and (A.47) to substitute for w_u and w_d in (A.33) and (A.34), respectively, (A.29), (A.32), and (A.49), we

obtain (after rearranging)

$$n = n_u - \nu_N \left[\frac{\varpi_d}{(1 - \alpha_{H,d}) \,\omega_{C,d}} \right] q_u,$$

$$n = n_d + \nu_N \left[\frac{\varpi_u}{(1 - \alpha_{H,d}) \,\omega_{C,d}} \right] q_u.$$

Substituting the expressions of n_u and n_d into the equations above yields, respectively,

$$\begin{split} n &= \frac{\gamma \omega_{G,u}}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_d} g_u + \frac{\gamma \tilde{\alpha}_{H,d} \omega_{G,d}}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_d} g_d \\ &+ \left\{ \frac{\tilde{\alpha}_{H,d} \left(1 - \gamma\right)}{1 - (1 - \tilde{\alpha}_{H,d}) \mu_d} \left[1 - \frac{\varpi_d}{1 - \alpha_{H,d}} + \frac{\alpha_{H,d} \mu_d}{(1 - \gamma) \left(1 - \alpha_{H,d}\right) \omega_{C,d}} - \frac{\mu_d \tilde{\alpha}_{H,d} + (1 - \gamma) \varpi_d \omega_{C,u}}{(1 - \gamma) \left(1 - \alpha_{H,d}\right) \tilde{\alpha}_{H,d} \omega_{C,d}} \right] \\ &- \nu_N \left[\frac{\varpi_d}{(1 - \alpha_{H,d}) \omega_{C,d}} \right] \right\} q_u, \end{split}$$

and

$$n = \frac{\gamma \omega_{G,d}}{\mu_d} g_d + \left\{ \frac{1-\gamma}{\mu_d} \left[1 - \frac{\varpi_d}{1-\alpha_{H,d}} + \frac{\alpha_{H,d}\mu_d}{(1-\gamma)\left(1-\alpha_{H,d}\right)\omega_{C,d}} \right] + \nu_N \left[\frac{\varpi_u}{(1-\alpha_{H,d})\omega_{C,d}} \right] \right\} q_u$$

Equating the two expressions yields, after some algebra

$$q_u = \frac{1}{\Upsilon} \left(\frac{\omega_{G,u}}{\mu_u} g_u - \frac{\omega_{G,d}}{\mu_d} g_d \right), \tag{A.53}$$

where $\Upsilon \equiv \frac{1}{\gamma} \left\{ (1-\gamma) \left[\frac{1}{\mu_d} - \frac{\varpi_d}{\varpi_d^e} \left(1 - \frac{\mu_d \ \omega_{C,u}}{\mu_u \ \omega_{C,d}} \right) \right] + \frac{\alpha_{H,d}\mu_u + \tilde{\alpha}_{H,d}\mu_d}{(1-\alpha_{H,d})\mu_u \omega_{C,d}} + \nu_N \left[\frac{1 - (1 - \tilde{\alpha}_{H,d})\mu_d}{(1-\alpha_{H,d})\omega_{C,d}} \right] \right\} > 0.$

In the case of perfect labor mobility (i.e., $\nu_N \to \infty$), $\Upsilon \to \infty$, so that $q_u = 0$, and $q_d = -\frac{\omega_{C,u}}{\omega_{C,d}}q_u = 0$. In the case of imperfect labor mobility (i.e., $\nu_N < \infty$), since $\mu_d, \mu_u > 0$, it is straightforward to show that

$$\begin{aligned} \frac{dq_u}{dg_u} &= \frac{1}{\Upsilon} \frac{\omega_{G,u}}{\mu_u} > 0, \\ \frac{dq_d}{dg_d} &= -\frac{\omega_{C,u}}{\omega_{C,d}} \frac{dq_u}{dg_d} = \frac{1}{\Upsilon} \frac{\omega_{C,u}}{\omega_{C,d}} \frac{\omega_{G,d}}{\mu_d} > 0 \end{aligned}$$

and

$$\begin{array}{ll} \displaystyle \frac{dq_u}{dg_d} & = & \displaystyle -\frac{1}{\Upsilon} \frac{\omega_{G,d}}{\mu_d} < 0, \\ \displaystyle \frac{dq_d}{dg_u} & = & \displaystyle -\frac{\omega_{C,u}}{\omega_{C,d}} \frac{dq_u}{dg_u} = \displaystyle -\frac{1}{\Upsilon} \frac{\omega_{C,u}}{\omega_{C,d}} \frac{\omega_{G,u}}{\mu_u} < 0. \end{array}$$

Proof of Proposition 3. Setting $\alpha_{H,s} = 0$ and $\vartheta_s = \vartheta$ implies that $\varpi_s = \varpi_s^e = \mu_s$ for s = u, d. Thus, (A.51) becomes

$$\frac{dy}{dg_s} = \gamma \omega_{G,s} + \left[\frac{\left(2-\gamma\right)\left(\mu_s-\omega_{C,s}\right)}{1-\omega_{C,s}}\right]\frac{dq_s}{dg_s},$$

Taking the derivative with respect to $\omega_{C,s}$ and using the definition of μ_s , we obtain

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \omega_{C,s}} = \frac{\gamma \left(2 - \gamma\right)}{1 - \omega_{C,s}} \left[\left(\frac{1 - \omega_{G,s}}{1 - \omega_{C,s}}\right) \frac{dq_s}{dg_s} + \left(\omega_{G,s} - \omega_{C,s}\right) \frac{\partial \left(\frac{dq_s}{dg_s}\right)}{\partial \omega_{C,s}} \right].$$
(A.54)

In addition, based on (A.53), we have

$$\frac{\partial \left(\frac{dq_s}{dg_s}\right)}{\partial \omega_{C,s}} = -\frac{\omega_{G,s} \left[\left(1 - \gamma\right) \Upsilon + \frac{\partial \Upsilon}{\partial \omega_{C,s}} \mu_s \right]}{\left(\Upsilon \mu_s\right)^2} = -\Theta \frac{dq_s}{dg_s},\tag{A.55}$$

where

$$\Theta \equiv \frac{\left(1-\gamma\right)\Upsilon + \frac{\partial\Upsilon}{\partial\omega_{C,s}}\mu_s}{\Upsilon\mu_s} > 0.$$

Substituting (A.55) into (A.54) yields

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \omega_{C,s}} = -\frac{\gamma \left(2-\gamma\right)}{1-\omega_{C,s}} \left[\frac{1-\omega_{G,s}}{1-\omega_{C,s}} + \left(\omega_{G,s}-\omega_{C,s}\right)\Theta\right] \frac{dq_s}{dg_s}$$

Proving the proposition amounts to showing that the term in square brackets is positive. This is always the case, for $\omega_{G,s} \geq \omega_{C,s}$. To prove it when $\omega_{G,s} < \omega_{C,s}$, we express $\frac{1-\omega_{G,s}}{1-\omega_{C,s}} + (\omega_{G,s} - \omega_{C,s})\Theta$ in terms of deep parameters, and rewrite it as

$$\frac{1 - \omega_{G,s}}{1 - \omega_{C,s}} + (\omega_{G,s} - \omega_{C,s}) \Theta = \underbrace{(1 - \gamma) \left[\frac{1 - \omega_{G,s}}{1 - \omega_{C,s}} \mu_s + (\omega_{G,s} - \omega_{C,s}) (1 - \gamma) \right] \left[\frac{(1 - \omega_{C,s}) \mu_s^2 + (1 - \mu_s)^2 \omega_{C,s}}{(1 - \mu_s) (1 - \omega_{C,s}) \mu_s} \right]}_{\Sigma_1} + \underbrace{\mu_s \left(\omega_{G,s} - \omega_{C,s} \right) (1 - \gamma) \left[\frac{(1 - \gamma)^2}{(1 - \mu_s)^2} + \frac{1 - \gamma}{(1 - \omega_{C,s})^2} \frac{(1 - \mu_s) \mu_s - (1 - \gamma) (1 - \omega_{C,s}) \omega_{C,s}}{\mu_s^2} \right]}_{\Sigma_2}}_{\Sigma_2} + \underbrace{\nu_N \frac{\left[(1 - \omega_{G,s}) \mu_s + (1 - \gamma) (\omega_{G,s} - \omega_{C,s}) (1 - \omega_{C,s}) (1 - \omega_{C,s} + \mu_s) \right]}{\Sigma_3}}_{\Sigma_3}.$$

Consider first the terms Σ_3 . Using the definition of μ_s , we can write it as

$$\Sigma_{3} = \nu_{N} \left\{ \frac{\left[(1-\gamma) \left(1 - \omega_{C,s} \right) + \gamma \left(1 - \omega_{G,s} \right) \right] \omega_{G,s} + \gamma \left(1 - \gamma \right) \left(\omega_{G,s} - \omega_{C,s} \right)^{2}}{\left(1 - \omega_{C,s} \right)^{2}} \right\},$$

which is always positive. It is therefore sufficient to prove that $\Sigma_1 + \Sigma_2 > 0$. After some algebra, this sum can be expressed as

$$\Sigma_{1} + \Sigma_{2} = \left[(1 - \omega_{G,s}) \,\mu_{s} + (1 - \gamma) \left(\omega_{G,s} - \omega_{C,s} \right) \Lambda \right] \left[\frac{(1 - \omega_{C,s}) \,\mu_{s}^{2} + (1 - \mu_{s})^{2} \,\omega_{C,s}}{(1 - \mu_{s}) \left(1 - \omega_{C,s} \right)^{2} \,\mu_{s}} \right],$$

where

$$\Lambda \equiv \frac{\left(2 - \mu_s - \gamma\right)\left(1 - \omega_{C,s}\right)^2 \mu_s^2 + \left(1 - \mu_s\right)^3 \left[\left(1 - \omega_{C,s}\right) \omega_{C,s} + \mu_s\right] - \left(1 - \gamma\right)\left(1 - \mu_s\right)^2 \left(1 - \omega_{C,s}\right) \omega_{C,s}}{\left(1 - \mu_s\right) \left[\left(1 - \omega_{C,s}\right) \mu_s^2 + \left(1 - \mu_s\right)^2 \omega_{C,s}\right]}$$

Since $\frac{(1-\omega_{C,s})\mu_s^2+(1-\mu_s)^2\omega_{C,s}}{(1-\mu_s)(1-\omega_{C,s})^2\mu_s} > 0$, we can focus on proving that $\mu_s (1-\omega_{G,s}) + (\omega_{G,s} - \omega_{C,s})(1-\gamma)\Lambda > 0$. We do this in two steps: first, we prove that $(1-\omega_{G,s})\mu_s + (1-\gamma)(\omega_{G,s} - \omega_{C,s}) > 0$, which implies that $\Lambda < 1$ is a sufficient condition for $\Sigma_1 + \Sigma_2$ to be strictly positive. The second step consists in proving that this sufficient condition holds. Step 1:

$$(1 - \omega_{G,s}) \mu_s + (1 - \gamma) (\omega_{G,s} - \omega_{C,s}) = (1 - \gamma) (1 - \omega_{G,s}) \omega_{C,s} + \gamma (1 - \omega_{G,s}) \omega_{G,s} + (1 - \gamma) (\omega_{G,s} - \omega_{C,s}) = - (1 - \gamma) \omega_{C,s} \omega_{G,s} + \gamma (1 - \omega_{G,s}) \omega_{G,s} + (1 - \gamma) \omega_{G,s} = (1 - \gamma) (1 - \omega_{C,s}) \omega_{G,s} + \gamma (1 - \omega_{G,s}) \omega_{G,s} > 0.$$

Step 2: Since $(1 - \mu_s) \left[(1 - \omega_{C,s}) \mu_s^2 + (1 - \mu_s)^2 \omega_{C,s} \right] > 0$, proving that $\Lambda < 1$ amounts to proving that the numerator of $\Lambda - 1$ is strictly negative. Denoting this object by Γ , we have

$$\begin{split} \Gamma &= (2 - \mu_s - \gamma) \left(1 - \omega_{C,s}\right)^2 \mu_s^2 + (1 - \mu_s)^3 \left[(1 - \omega_{C,s}) \,\omega_{C,s} + \mu_s \right] - (1 - \gamma) \left(1 - \mu_s\right)^2 \left(1 - \omega_{C,s}\right) \omega_{C,s} \\ &- (1 - \mu_s) \left[(1 - \omega_{C,s}) \,\mu_s^2 + (1 - \mu_s)^2 \,\omega_{C,s} \right] \\ &= (1 - \mu_s)^2 \left[(1 - \mu_s) \left(\mu_s - \omega_{C,s}^2 \right) - (1 - \gamma) \left(1 - \omega_{C,s}\right) \omega_{C,s} \right] + \mu_s^2 \left(1 - \omega_{C,s}\right) \left[(1 - \gamma) \left(1 - \omega_{C,s}\right) - \omega_{C,s} \left(1 - \mu_s\right) \right] \\ &= (1 - \gamma) \left(1 - \omega_{C,s}\right) \left[(1 - \omega_{C,s}) \,\mu_s^2 - (1 - \mu_s)^2 \,\omega_{C,s} \right] + (1 - \mu_s) \left[(1 - \mu_s)^2 \left(\mu_s - \omega_{C,s}^2 \right) - (\omega_{C,s} - \omega_{C,s}^2) \mu_s^2 \right]. \end{split}$$

If $(1 - \omega_{C,s}) \mu_s^2 - (1 - \mu_s)^2 \omega_{C,s} > 0$, then we can write

$$\begin{split} \Gamma &< (1 - \omega_{C,s}) \left[(1 - \omega_{C,s}) \, \mu_s^2 - (1 - \mu_s)^2 \, \omega_{C,s} \right] + (1 - \mu_s) \left[(1 - \mu_s)^2 \, (\mu_s - \omega_{C,s}^2) - (\omega_{C,s} - \omega_{C,s}^2) \mu_s^2 \right] \\ &= (1 - \mu_s)^2 \left[(\mu_s - \omega_{C,s}^2) \, (1 - \mu_s) - (\omega_{C,s} - \omega_{C,s}^2) \right] + \mu_s^2 \, (1 - \omega_{C,s}) \left[(1 - \omega_{C,s}) - (1 - \mu_s) \, \omega_{C,s} \right] \\ &< (1 - \mu_s)^2 \left[(\omega_{C,s} - \omega_{C,s}^2) \, (1 - \mu_s) - (\omega_{C,s} - \omega_{C,s}^2) \right] + \mu_s^2 \, (1 - \omega_{C,s}) \left[(1 - \omega_{C,s}) - (1 - \mu_s) \, \omega_{C,s} \right] \\ &= \mu_s \, (1 - \omega_{C,s}) \left[- (1 - \mu_s)^2 \, \omega_{C,s} + (1 - \omega_{C,s}) \, \mu_s - (1 - \mu_s) \, \mu_s \omega_{C,s} \right] \\ &= (1 - \omega_{C,s}) \, (\mu_s - \omega_{C,s}) \, \mu_s < 0, \end{split}$$

where the last inequality follows from the fact that $\mu_s < \omega_{C,s}$ when $\omega_{G,s} < \omega_{C,s}$. In turn, this inequality implies that $(1 - \mu_s) \left[(1 - \mu_s)^2 (\mu_s - \omega_{C,s}^2) - (\omega_{C,s} - \omega_{C,s}^2) \mu_s^2 \right] < 0$. If $(1 - \omega_{C,s}) \mu_s^2 - (1 - \mu_s)^2 \omega_{C,s} < 0$, then it follows immediately that $\Gamma < 0$.

Proof of Proposition 4. Part (i) To isolate the role of the sectoral markup, ϑ_s , we assume that the two sectors are

otherwise identical by setting
$$\omega_{C,s} = \omega_{G,s} = 1/2$$
 and $\alpha_{H,s} = 0$ for $s = u, d$. Thus, (A.51) becomes

$$\frac{dy}{dg_s} = \frac{\gamma}{2} - 2\left(1 - \gamma\right)\left(\varpi_s^e - \varpi_s\right)\frac{dq_s}{dg_s}$$

where

$$\overline{\omega}_s^e - \overline{\omega}_s = \frac{\vartheta_s}{\sum_{s=u,d} \vartheta_s} - \frac{1}{2}.$$

Taking the derivative with respect to ϑ_s yields

$$\frac{\partial \left(\frac{dy}{dg_s}\right)}{\partial \vartheta_s} = -2\left(1-\gamma\right) \left\{ \left[\frac{\sum_{s=u,d} \vartheta_s - \vartheta_s}{\left(\sum_{s=u,d} \vartheta_s\right)^2}\right] \frac{dq_s}{dg_s} + \left[\frac{\vartheta_s}{\sum_{s=u,d} \vartheta_s} - \frac{1}{2}\right] \frac{\partial \left(\frac{dq_s}{dg_s}\right)}{\partial \vartheta_s} \right\} < 0.$$

since $\frac{dq_s}{dg_s} > 0$ and $\frac{\partial \left(\frac{dq_s}{dg_s}\right)}{\partial \vartheta_s} = 0$ for s = u, d.

Part (ii) To isolate the role of the sectoral labor intensity, $1 - \alpha_{H,s}$, while maintaining the assumption of constant returns to scale in production, it would be natural to consider an economy with a roundabout production structure whereby the two sectors only use their own goods as intermediate inputs. Assuming that the two sectors are otherwise symmetric, one could study the role of $1 - \alpha_{H,s}$ by varying it in one of the two sectors. Although our simple economy does not allow for an identity Input-Output matrix, the exercise just described is equivalent to varying the labor intensity of sector uwhile assuming that sector d uses no intermediate inputs. This is the argument underlying the proof below.

Setting $\omega_{C,s} = \omega_{G,s} = 1/2$ and $\vartheta_s = \vartheta$ for s = u, d, (A.51) becomes for s = u

$$\frac{dy}{dg_u} = \frac{\gamma}{2} - \left[\frac{2\left(1-\gamma\right)\left(\varpi_u^e - \varpi_u\right)}{1-\alpha_{H,d}}\right]\frac{dq_u}{dg_u},$$

where

$$\varpi_u^e - \varpi_u = \frac{1 + \alpha_{H,d}}{2} - \frac{(1 - \alpha_{H,u})\left(\vartheta - \alpha_{H,d}\right)\left(1 + \tilde{\alpha}_{H,d}\right)}{\left(1 - \alpha_{H,u}\right)\left(\vartheta - \alpha_{H,d}\right)\left(1 + \tilde{\alpha}_{H,d}\right) + \left(1 - \alpha_{H,d}\right)\left(\vartheta - \alpha_{H,u}\right)\left(1 - \tilde{\alpha}_{H,d}\right)}$$

Taking the derivative with respect to $\alpha_{H,u}$ yields

$$\frac{\partial \left(\frac{dy}{dg_u}\right)}{\partial \left(1-\alpha_{H,u}\right)} = -\frac{\partial \left(\frac{dy}{dg_u}\right)}{\partial \alpha_{H,u}} = \frac{2\left(1-\gamma\right)}{1-\alpha_{H,d}} \left\{ \frac{\partial \left(\varpi_u^e - \varpi_u\right)}{\partial \alpha_{H,u}} \frac{dq_u}{dg_u} + \left(\varpi_u^e - \varpi_u\right) \frac{\partial \left(\frac{dq_u}{dg_u}\right)}{\partial \alpha_{H,u}} \right\},$$

Since

$$\frac{\partial \left(\varpi_{u}^{e} - \varpi_{u}\right)}{\partial \alpha_{H,u}} = -\frac{\left(1 + \tilde{\alpha}_{H,d}\right)\left(1 - \tilde{\alpha}_{H,d}\right)\left(\vartheta - \alpha_{H,d}\right)\left(1 - \alpha_{H,d}\right)\left(1 - \vartheta\right)}{\left[\left(1 - \alpha_{H,u}\right)\left(\vartheta - \alpha_{H,d}\right)\left(1 + \tilde{\alpha}_{H,d}\right) + \left(1 - \alpha_{H,d}\right)\left(\vartheta - \alpha_{H,u}\right)\left(1 - \tilde{\alpha}_{H,d}\right)\right]^{2}} > 0$$

and

$$\frac{\partial \left(\frac{dq_u}{dg_u}\right)}{\partial \alpha_{H,u}} = 0,$$

we have

$$\frac{\partial \left(\frac{dy}{dg_u}\right)}{\partial \left(1-\alpha_{H,u}\right)} > 0$$

Part (iii) To isolate the role of the position in the network, we assume that the two sectors are otherwise identical by setting $\omega_{C,s} = \omega_{G,s} = 1/2$, $\alpha_{H,s} = \alpha_H$, and $\vartheta_s = \vartheta$ for s = u, d, which yields

$$\begin{aligned} \varpi_u^e - \varpi_u &= \frac{(1 - \vartheta^{-1})\alpha_H}{2}, \\ \varpi_d^e - \varpi_d &= -\frac{(1 - \vartheta^{-1})\alpha_H}{2}. \end{aligned}$$

Thus, (A.51) implies

$$\begin{aligned} \frac{dy}{dg_u} &= \frac{\gamma}{2} - \frac{1}{2} \left[\frac{(1-\gamma)\left(1-\vartheta^{-1}\right)\alpha_H}{1-\alpha_H} \right] \frac{dq_u}{dg_u}, \\ \frac{dy}{dg_d} &= \frac{\gamma}{2} + \frac{1}{2} \left[\frac{(1-\gamma)\left(1-\vartheta^{-1}\right)\alpha_H}{1-\alpha_H} \right] \frac{dq_d}{dg_d}. \end{aligned}$$

Since $(1 - \vartheta^{-1}) > 0$ and $\frac{dq_s}{dg_s} > 0$ for s = u, d, it follows that $\frac{dy}{dg_d} > \frac{dy}{dg_u}$.

B Full Description of the Quantitative Model



Figure B.1: Structure of the Quantitative Model.

B.1 Households

The economy is populated by an infinitely-lived representative household that has preferences over aggregate consumption, C_t , the sum of government purchases from all sectors, G_t , and aggregate labor, N_t , so that its expected lifetime utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{X_t^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right\},\tag{B.1}$$

$$X_{t} = \left[\zeta_{t}^{\frac{1}{\xi}} C_{t}^{\frac{\xi-1}{\xi}} + (1-\zeta)^{\frac{1}{\xi}} G_{t}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}},$$
(B.2)

where β is the subjective time discount factor, σ captures the degree of risk aversion, θ is a preference parameter that affects the disutility of labor, and η is the inverse of the Frisch elasticity of labor supply. As in Bouakez and Rebei (2007), preferences are non-separable in consumption and government services. In this specification, the parameter ζ denotes the weight of aggregate consumption in total consumption services, X_t , whereas ξ is the elasticity of substitution between aggregate consumption and aggregate government services.

The household enters period t with a stock of nominal bonds, B_t , and a stock of physical capital, K_t . During the period, it receives the principal and the interest on its bonds holdings – with R_t denoting the gross nominal interest rate – provides labor and rents physical capital to the intermediate-good producers in exchange of a nominal wage rate, W_t , and a nominal rental rate, $R_{K,t}$. It also receives nominal profits from intermediate-good producers in all sectors, $\sum_{s=1}^{S} D_{s,t}$, and pays a nominal lump-sum tax, T_t , to the government. The household purchases a bundle of consumption goods at price $P_{C,t}$, and one of investment goods, I_t , at price $P_{I,t}$, and allocates its remaining income to the purchase of

new bonds. Its budget constraint is therefore given by

$$P_{C,t}C_t + P_{I,t}I_t + B_{t+1} + T_t = W_t N_t + R_{K,t}K_t + B_t R_{t-1} + \sum_{s=1}^{S} D_{s,t}.$$
(B.3)

Investment is subject to convex adjustment costs, so that the stock of physical capital evolves over time according to

$$K_{t+1} = (1-\delta) K_t + I_t \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right],$$
(B.4)

where δ is the depreciation rate and Ω captures the magnitude of the adjustment cost. The household chooses C_t , N_t , I_t , K_{t+1} , and B_{t+1} to maximize life-time utility (B.1) subject to the budget constraint (B.3), the accumulation equation (B.4), and a no-Ponzi-game condition.

The total amount of labor provided by the household is a CES function of the labor supplied to each sector, that is

$$N_{t} = \left[\sum_{s=1}^{S} \omega_{N,s}^{-\frac{1}{\nu_{N}}} N_{s,t}^{\frac{1+\nu_{N}}{\nu_{N}}}\right]^{\frac{\nu_{N}}{1+\nu_{N}}},$$
(B.5)

where $\omega_{N,s}$ is the weight attached to labor provided to sector s, and ν_N denotes (the absolute value of) the elasticity of substitution of labor across sectors. When $\nu_N \to \infty$, labor is perfectly mobile and nominal wages are equalized across sectors. Instead, as long as $\nu_N < \infty$, labor is imperfectly mobile and sectoral wages can differ. The nominal wage rate is defined as a function of the nominal sectoral wages, $W_{s,t}$, and reads as

$$W_{t} = \left[\sum_{s=1}^{S} \omega_{N,s} W_{s,t}^{1+\nu_{N}}\right]^{\frac{1}{1+\nu_{N}}}.$$
(B.6)

Analogously, the total amount of physical capital is given by the CES function

$$K_{t} = \left[\sum_{s=1}^{S} \omega_{K,s}^{-\frac{1}{\nu_{K}}} K_{s,t}^{\frac{1+\nu_{K}}{\nu_{K}}}\right]^{\frac{\nu_{K}}{1+\nu_{K}}},$$
(B.7)

where $\omega_{K,s}$ is the weight attached to capital provided to sector s, and ν_K is (the absolute value of) the elasticity of substitution of capital across sectors. The aggregate nominal rental rate of capital is defined as

$$R_{K,t} = \left[\sum_{s=1}^{S} \omega_{K,s} R_{K,s,t}^{1+\nu_K}\right]^{\frac{1}{1+\nu_K}},$$
(B.8)

where $R_{K,s,t}$ is the nominal rental rate of capital in sector s. equilibrium, capital is allocated across sectors such that the following first-order conditions hold

$$K_{s,t} = \omega_{K,s} \left(\frac{R_{K,s,t}}{R_{K,t}}\right)^{\nu_K} K_t, \qquad s = 1, \dots, S.$$
(B.9)

B.2 Firms

In each sector, there is a continuum of producers that assemble differentiated varieties of output using labor, capital, and a bundle of intermediate inputs. These varieties are then aggregated into a single good in each sector by a representative wholesaler. The goods produced by the S representative wholesalers are then purchased by the retailers, who assemble them into consumption and investment bundles sold to the households, and intermediate-input bundles sold to the producers.

B.2.1 Producers

In each sector, there is a continuum of monopolistically competitive producers, indexed by $j \in [0, 1]$, that use labor, capital, and a bundle of intermediate inputs to assemble a differentiated variety using the Cobb-Douglas technology

$$Z_{s,t}^{j} = \left(N_{s,t}^{j} K_{s,t}^{\alpha_{N,s}} K_{s,t}^{j}\right)^{1-\alpha_{H,s}} H_{s,t}^{j} K_{s,t}^{\alpha_{H,s}},$$
(B.10)

where $Z_{s,t}^{j}$ is the gross output of the variety of producer j, $N_{s,t}^{j}$, $K_{s,t}^{j}$, and $H_{s,t}^{j}$ denote labor, capital, and the bundle of intermediate inputs used by this producer. The parameters $\alpha_{N,s}$ and $\alpha_{H,s}$ are the value-added labor intensity and the gross-output intensity of intermediate inputs, respectively.

As producer j sells its output $Z_{s,t}^{j}$ at price $P_{s,t}^{j}$ to the wholesalers, hires labor at the wage $W_{s,t}$, rents capital at the rate $R_{K,s,t}$, and purchases intermediate inputs at the price $P_{H,s,t}$, its nominal profits equal

$$D_{s,t}^{j}\left(P_{s,t}^{j}\right) = P_{s,t}^{j}Z_{s,t}^{j} - W_{s,t}N_{s,t}^{j} - R_{K,s,t}K_{s,t}^{j} - P_{H,s,t}H_{s,t}^{j}.$$
(B.11)

Producers set their price according to a Calvo-type pricing protocol. The Calvo probability that the price remains fixed from one period to the next is constant and identical across producers within the same sector. However, we allow this probability to differ across sectors, and denote it by ϕ_s . By the law of large numbers, a fraction $1 - \phi_s$ of producers are able to reset their prices in each period.

B.2.2 Wholesalers

In each sector, perfectly competitive wholes alers aggregate the different varieties supplied by the producers into a single final good. The representative wholesaler in sector s has the following CES production technology:

$$Z_{s,t} = \left[\int_0^1 Z_{s,t}^j \frac{\epsilon_s - 1}{\epsilon_s} dj \right]^{\frac{\epsilon_s}{\epsilon_s - 1}}, \tag{B.12}$$

where $Z_{s,t}$ is the output of sector s, and ϵ_s is the elasticity of substitution across varieties within sector s. The price of the final good s is then given by

$$P_{s,t} = \left[\int_{0}^{1} P_{s,t}^{j} {}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}.$$
(B.13)

and the problem of the representative wholes aler in sector \boldsymbol{s} reads as

$$\max_{\substack{Z_{s,t}^{j}\\ s,t}} P_{s,t} Z_{s,t} - \int_{0}^{1} P_{s,t}^{j} Z_{s,t}^{j} dj$$

s.t.
$$Z_{s,t} = \left[\int_{0}^{1} Z_{s,t}^{j} \frac{\epsilon - 1}{\epsilon} dj \right]^{\frac{\epsilon}{\epsilon - 1}}$$

which implies the following first-order conditions:

$$Z_{s,t}^{j} = \left(\frac{P_{s,t}^{j}}{P_{s,t}}\right)^{-\epsilon} Z_{s,t}, \qquad j \in [0,1], \ s = 1, \dots, S.$$
(B.14)

The final good of sector s is sold to consumption, investment, and intermediate-input retailers, as well as to the fiscal authority. This yields the following market-clearing condition:

$$Z_{s,t} = C_{s,t} + I_{s,t} + \sum_{x=1}^{S} H_{x,s,t} + G_{s,t},$$
(B.15)

where $C_{s,t}$ and $I_{s,t}$ denote, respectively, the retailer's purchase of consumption and investment goods from the wholesaler of sector s, $H_{x,s,t}$ denotes the intermediate inputs produced by sector s and used in the production of sector x, and $G_{s,t}$ denotes government purchases from sector s.

B.2.3 Consumption-good retailers

Perfectly competitive consumption-good retailers purchase goods from the wholesalers of each sector and assemble them into a consumption bundle sold to households. The representative consumption-good retailer uses the following CES technology:

$$C_{t} = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_{C}}} C_{s,t}^{\frac{\nu_{C}-1}{\nu_{C}}}\right]^{\frac{\nu_{C}}{\nu_{C}-1}},$$
(B.16)

where ν_C is the elasticity of substitution of consumption across sectors, and $\omega_{C,s}$ denotes the weight of good s in the consumption bundle, such that $\sum_{s=1}^{S} \omega_{C,s} = 1$. The consumption bundle is sold to the households at the equilibrium price $P_{C,t}$, defined as

$$P_{C,t} = \left[\sum_{s=1}^{S} \omega_{C,s} P_{s,t}^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}}.$$
(B.17)

Therefore, the consumption-good retailer solves the following problem:

$$\max_{C_{s,t}} P_{C,t}C_t - \sum_{s=1}^{S} P_{s,t}C_{s,t}$$

s.t. $C_t = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_C}} C_{s,t}^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}$

which yields the following first-order conditions:

$$C_{s,t} = \omega_{C,s} \left(\frac{P_{s,t}}{P_{C,t}}\right)^{-\nu_C} C_t, \qquad s = 1, \dots, S.$$
(B.18)

B.2.4 Investment-good retailers

Investment-good retailers behave analogously to the consumption-good retailers. The representative investment-good retailer buys goods from the representative wholesaler of each sector and assembles them into an investment bundle using the CES technology

$$I_t = \left[\sum_{s=1}^{S} \omega_{I,s}^{\frac{1}{\nu_I}} I_{s,t}^{\frac{\nu_I - 1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I - 1}},\tag{B.19}$$

where ν_I is the elasticity of substitution of investment across sectors, and $\omega_{I,s}$ denotes the weight of good s in the investment bundle, such that $\sum_{s=1}^{S} \omega_{I,s} = 1$. The investment bundle is sold to the households at the equilibrium price $P_{I,t}$, defined as

$$P_{I,t} = \left[\sum_{s=1}^{S} \omega_{I,s} P_{s,t}^{1-\nu_{I}}\right]^{\frac{1}{1-\nu_{I}}}.$$
(B.20)

and the first-order conditions associated with the retailer's optimization problem are given by

$$I_{s,t} = \omega_{I,s} \left(\frac{P_{s,t}}{P_{I,t}}\right)^{-\nu_I} I_t, \qquad s = 1, \dots, S.$$
(B.21)

B.2.5 Intermediate-input retailers

Perfectly competitive intermediate-input retailers transform the goods assembled by the wholesale producers of all sectors into a bundle of intermediate inputs destined exclusively to the producers of a specific sector. The representative intermediate-input retailer that sells exclusively to sector s produces the bundle $H_{s,t}$ using the CES technology

$$H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x}^{\frac{1}{\nu_{H}}} H_{s,x,t}^{\frac{\nu_{H}-1}{\nu_{H}}}\right]^{\frac{\nu_{H}}{\nu_{H}-1}},$$
(B.22)

where $H_{s,x,t}$ is the quantity of goods purchased from the wholesaler of sector x, ν_H is the elasticity of substitution of intermediate inputs across sectors, and $\omega_{H,s,x}$ is the weight of the intermediate inputs produced by sector x in the total

amount of intermediate inputs used by firms in sector s, such that $\sum_{x=1}^{S} \omega_{H,s,x} = 1$. The intermediate-input bundle is sold to firms in sector s at the equilibrium price $P_{H,s,t}$, which satisfies

$$P_{H,s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x} P_{x,t}^{1-\nu_H}\right]^{\frac{1}{1-\nu_H}}.$$
(B.23)

The problem of this intermediate-input retailer, therefore, is

$$\max_{H_{s,x,t}} P_{H,s,t} H_{s,t} - \sum_{x=1}^{S} P_{x,t} H_{s,x,t}$$

s.t.
$$H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x}^{\frac{1}{\nu_{H}}} H_{s,x,t}^{\frac{\nu_{H}-1}{\nu_{H}}} \right]^{\frac{\nu_{H}}{\nu_{H}-1}}$$

which implies the following first-order conditions:

$$H_{s,x,t} = \omega_{H,s,x} \left(\frac{P_{x,t}}{P_{H,s,t}}\right)^{-\nu_H} H_{s,t}, \qquad s, x = 1, \dots, S.$$
(B.24)

B.3 Government

The government consists of a monetary and a fiscal authority. The monetary authority sets the nominal interest rate, R_t , according to the Taylor rule

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\varphi_R} \left[(1+\pi_t)^{\varphi_\Pi} \left(\frac{Y_t}{Y_t^{\text{flex}}}\right)^{\varphi_Y} \right]^{1-\varphi_R},\tag{B.25}$$

where R^* denotes the steady-state nominal interest rate, φ_R is the degree of interest rate inertia, Y_t is aggregate real value added of a counterfactual economy with fully flexible prices, φ_{Π} and φ_Y measure the degree to which the monetary authority adjusts the nominal interest rate in response to changes in aggregate inflation π_t and the output gap $\frac{Y_t}{Y_t^{\text{flex}}}$, respectively. The aggregate inflation is derived over the GDP deflator P_t , that is, $\pi_t = \frac{P_t}{P_{t-1}} - 1$.

Government purchases from sector s are governed by the following auto-regressive process:

$$\log G_{s,t} = (1-\rho)\log G_s^* + \rho \log G_{s,t-1} + v_{s,t}, \tag{B.26}$$

where ρ measures the persistence of the process. Sectoral government spending changes over time following the realizations of the unique source of uncertainty in the model: sectoral government spending shocks, $v_{s,t}$, which are zero-mean innovations. Once the spending shocks are realized, the government purchases goods from the representative wholesaler at price $P_{s,t}$. Government purchases are financed through lump-sum taxes paid by the household, which implies the following budget constraint for the government:

$$\sum_{s=1}^{S} P_{s,t} G_{s,t} = T_t.$$
(B.27)

B.4 Aggregation

Let $Y_{s,t}$ denote the nominal value added of producer j in sector s, defined as the value of gross output produced by the producer less the cost of the intermediate inputs it uses. That is,

$$\mathcal{Y}_{s,t}^{j} = P_{s,t}^{j} Z_{s,t}^{j} - P_{H,s,t} H_{s,t}^{j}.$$
(B.28)

Aggregating the nominal value added of all the producers in sector s yields

$$\mathcal{Y}_{s,t} = \int_0^1 \mathcal{Y}_{s,t}^j dj = P_{s,t} Z_{s,t} - P_{H,s,t} H_{s,t}.$$
(B.29)

Moreover, summing up nominal dividends across firms within sector s yields

$$D_{s,t} = \int_0^1 D_{s,t}^j dj = P_{s,t} Z_{s,t} - W_{s,t} N_{s,t} - R_{K,s,t} K_{s,t} - P_{H,s,t} H_{s,t}$$

= $\mathcal{Y}_{s,t} - W_{s,t} N_{s,t} - R_{K,s,t} K_{s,t}.$ (B.30)

Aggregating nominal dividends across sectors and substituting into the households' budget constraint (B.3), we obtain⁴¹

$$\mathcal{Y}_{t} = \sum_{s=1}^{S} \mathcal{Y}_{s,t} = P_{C,t}C_{t} + P_{I,t}I_{t} + \sum_{s=1}^{S} P_{s,t}G_{s,t}.$$
(B.31)

Equation (B.31) states that aggregate nominal value added, Y_t , equals the sum of the nominal values of consumption, investment, and government spending.

Aggregate real value added is defined as the ratio between aggregate nominal value added and the GDP deflator:

$$Y_t = \frac{\mathcal{Y}_t}{P_t}.\tag{B.32}$$

Using an analogous definition for sectoral real value added, $Y_{s,t}$, aggregate real value added satisfies the following identity:

$$Y_t = \frac{\sum_{s=1}^{S} \mathcal{Y}_{s,t}}{P_t} = \sum_{s=1}^{S} Y_{s,t}.$$
 (B.33)

B.5 Nesting the stylized economy

The multi-sector model collapses to the stylized two-sector economy of Section 2 under the following parameter restrictions:

- 1. The economy consists of two sectors: S = 2, and s = u, d;
- 2. The upstream sector (u) supplies all the intermediate inputs, whereas the downstream sector (d) demands intermediate inputs, but provides none: $\omega_{H,u,u} = \omega_{H,d,u} = 1$;
- 3. Cobb-Douglas consumption-good aggregator: $\nu_C = 1$;
- 4. No capital in the production function: $\alpha_{N,u} = \alpha_{N,d} = 1$;
- 5. Equal gross-output factor intensities across sectors: $\alpha_{H,u} = \alpha_{H,d} = \alpha_H$;
- 6. Fully flexible prices: $\phi_u = \phi_d = 0$;
- 7. Equal steady-state ratio of the sectoral contribution to government spending to aggregate value added: $\frac{Q_u^* G_u^*}{Y^*} = \frac{Q_d^* G_d^*}{Y^*};$
- 8. Households receive no utility from government spending: $\zeta = 1$;
- 9. Logarithmic preference over consumption: $\sigma = 1$;
- 10. Infinite Frisch elasticity of labor supply: $\eta = 0$.

⁴¹To derive this equation, we have used the market clearing conditions $N_{s,t} = \int_0^1 N_{s,t}^j dj$, $K_{s,t} = \int_0^1 K_{s,t}^j dj$, and $H_{s,t} = \int_0^1 H_{s,t}^j dj$, the government budget constraint (B.27), the retailers' zero-profit conditions $\sum_s W_{s,t} N_{s,t} = W_t N_t$ and $\sum_s R_{K,s,t} K_{s,t} = R_{K,t} K_t$, as well as the fact that the net supply of private bonds equals zero in equilibrium: $B_t = 0, \forall t$.

C More on the Calibration of the Quantitative Model

This section presents further information on the calibration of the quantitative model. Tables C.1 - C.3 report the list of the 57 production sectors we consider. This level of disaggregation roughly corresponds to the three-digit level of the NAICS codes.

C.1 Parameter values

In what follows, we discuss the calibration of the parameters that are common to all sectors and we have not mentioned in Section 2.2.2. All these values are also reported in Table C.4, together with the target or the source that disciplines our calibration choice. We calibrate the time discount factor to $\beta = 0.995$, such that the annual steady-state nominal interest rate equals 2 percent, while the risk-aversion parameter is set to the standard value of $\sigma = 2$. Moreover, we choose $\eta = 1.25$, such that the Frisch elasticity equals 0.8, in line with the estimate of the labor supply elasticity derived by Chetty et al. (2013). We set $\theta = 41.57$, such that the steady-state level of total hours, N, equals 0.33. We choose the elasticity of substitution between consumption and government spending to be $\xi = 0.3$, in line with the estimates of Bouakez and Rebei (2007) and Sims and Wolff (2018),⁴² and the relative weight of consumption to be $\zeta = 0.7$, which corresponds to the ratio of the nominal value of consumption expenditures over the sum of consumption and government expenditures.

We set $\delta = 0.025$, which implies that physical capital depreciates by 10 percent on an annual basis. We calibrate the investment-adjustment-cost parameter such that the response of aggregate inflation to a common government spending shock peaks after eight quarters, in line with the empirical evidence of Blanchard and Perotti (2002). Accordingly, we set $\Omega = 25$.

As for the Taylor rule, we use the estimates of Clarida, Gali and Gertler (2000): we set the degree of interest-rate inertia to $\varphi_R = 0.8$, and the responsiveness to changes in the inflation rate and to the output gap to $\varphi_{\Pi} = 1.5$ and $\varphi_Y = 0.2$, respectively.

Finally, the tables that report the parameters that vary across sectors (i.e., the contribution to the final consumption good, the contribution to the final investment good, the contribution to government spending, the entire Input-Output matrix, the factor intensities, and the degree of price rigidity) are available upon request.

⁴²Consumption and government spending are therefore Edgeworth complements in utility. This assumption is supported by the empirical evidence reported by Fève, Matheron and Sahuc (2013) and Leeper, Traum and Walker (2017).

- 1 Farms
- 2 Forestry, fishing, and related activities
- 3 Mining
- 4 Utilities
- 5 Construction
- 6 Wood products
- 7 Nonmetallic mineral products
- 8 Primary metals
- 9 Fabricated metal products
- 10 Machinery
- 11 Computer and electronic products
- 12 Electrical equipment, appliances, and components
- 13 Motor vehicles, bodies and trailers, and parts
- 14 Other transportation equipment
- 15 Furniture and related products
- 16 Miscellaneous manufacturing
- 17 Food and beverage and tobacco products
- 18 Textile mills and textile product mills
- 19 Apparel and leather and allied products
- 20 Paper products

Table C.2: Sectors 21-40.

- 21 Printing and related support activities
- 22 Petroleum and coal products
- 23 Chemical products
- 24 Plastics and rubber products
- 25 Wholesale trade
- 26 Motor vehicle and parts dealers
- 27 Food and beverage stores
- 28 General merchandise stores
- 29 Other retail
- 30 Air transportation
- 31 Rail transportation
- 32 Water transportation
- 33 Truck transportation
- 34 Transit and ground passenger transportation
- 35 Pipeline transportation
- 36 Other transportation and support activities
- 37 Warehousing and storage
- 38 Publishing industries, except internet (includes software)
- 39 Motion picture and sound recording industries
- 40 Broadcasting and telecommunications

- 41 Data processing, internet publishing, and other information services
- 42 Legal services
- 43 Computer systems design and related services
- 44 Miscellaneous professional, scientific, and technical services
- 45 Management of companies and enterprises
- 46 Administrative and support services
- 47 Waste management and remediation services
- 48 Educational services
- 49 Ambulatory health care services
- 50 Hospitals
- 51 Nursing and residential care facilities
- 52 Social assistance
- 53 Performing arts, spectator sports, museums, and related activities
- 54 Amusements, gambling, and recreation industries
- 55 Accommodation
- 56 Food services and drinking places
- 57 Other services, except government

Table C.4: Calibration of Economy-Wide Parameters.

Parameter	Target/Source
$\beta = .995$	2%Steady-State Annual Interest Rate R
$\sigma = 2$	Standard Value
$\theta = 41.01$	0.33 Steady-State Total Hours N
$\eta = 1/0.8$	Chetty et al. (2013)
$\xi = 0.3$	Bouakez and Rebei (2007), Sims and Wolff (2018)
$\zeta = 0.7$	Ratio of nominal value of consumption expenditures over the sum of consumption and government expenditures
$\delta = 0.025$	10% Annual Depreciation Rate
$\Omega = 20$	8 Quarters Peak Response of Investment
$\nu_C = 2$	Hobijn and Nechio (2019)
$\nu_I = 2$	$\nu_I = \nu_C$
$\nu_H = 0.1$	Barrot and Sauvagnat (2016), Atalay (2017), Boehm, Flaaen and Pandalai-Nayar (2019)
$\nu_N = 1$	Horvath (2000)
$\nu_K = 1$	$\nu_K = \nu_N$
$\epsilon = 4$	33% Steady-State Mark-Up
$\varphi_R=0.8$	Clarida, Gali and Gertler (2000)
$\varphi_{\Pi}=1.5$	Clarida, Gali and Gertler (2000)
$\varphi_Y=0.2$	Clarida, Gali and Gertler (2000)
$\rho = 0.9$	Standard Value

C.2 Validating the calibration

We assess the empirical plausibility of our calibration strategy by evaluating the model's ability to account for the crossindustry elasticity of output to government purchases, in the spirit of Nakamura and Steinsson (2014, 2018), Beraja, Hurst and Ospina (2019), and Jones, Midrigan and Philippon (2023).

To do so, we build a panel of sectoral value added and government defense spending at an annual frequency, from 1958 to 2018, using data from the U.S. Bureau of Economic Analysis (BEA). The panel is constructed at the same level of sectoral disaggregation considered in the model,⁴³ and is used to estimate the following regression:

$$\Delta \log Y_{s,t} = \beta_1 \Delta \log \tilde{G}_{s,t} + \beta_2 \Delta \log Y_{s,t-1} + \beta_3 \Delta \log \tilde{G}_{s,t-1} + \delta_s + \delta_t + \epsilon_{s,t}, \tag{C.34}$$

where the dependent variable, $\Delta \log Y_{s,t}$, is the log-change in the value added of sector s, and $\Delta \log \tilde{G}_{s,t}$ is the defense spending shock in sector s. The regression also includes lagged values of sectoral value added and government spending, as well as industry fixed effects, δ_s , and year fixed effects, δ_t .

Since sectoral defense purchases, $G_{s,t}$, could depend on the economic conditions of the recipient industry, we identify the sectoral shocks by assuming that the allocation of aggregate defense spending across sectors remains constant over time. Specifically, we closely follow Nekarda and Ramey (2011) and Acemoglu, Akcigit and Kerr (2016) and construct the series of defense spending shocks as

$$\Delta \log \tilde{G}_{s,t} = \theta_s \times \Delta \log G_t, \tag{C.35}$$

where G_t is the aggregate series of real defense spending, and θ_s is the sample average of the ratio between sectoral defense spending in sector s and sectoral total shipments, $Z_{s,t}$, that is

$$\theta_s = \frac{1}{T} \sum_{t=1}^T \frac{G_{s,t}}{Z_{s,t}}.$$
(C.36)

While the common practice of using military spending as a proxy for government expenditure relies on the premise that the U.S. do not embark in a war because national value added is low (Barro and Redlick, 2011; Ramey, 2011), our approach implies a much weaker identifying condition, requiring that the U.S. do not embark in a war when the output of a given sector is lower than that of all the other industries. In this setting, the coefficient β_1 denotes the elasticity of the value added of sector s — relative to the elasticity of the value added of all the other industries — to a 1 percent increase in real defense spending in sector s.

Using the baseline economy, we simulate — and aggregate at an annual frequency — an analogous artificial panel of sectoral value added and government defense spending, and construct a series of defense spending shocks according to Equation (C.35). We then use the artificial data to estimate Equation (C.34). Our validation exercise therefore consists in comparing the model-based estimate of β_1 with its data-based counterpart.

Table C.5 reports the estimation results. The model-based estimate of β_1 (0.79) is extremely close to its data-based counterpart (0.75), indicating that the model successfully accounts for the cross-industry output effects of government spending shocks measured in the data.

 $^{^{43}}$ More precisely, the constructed panel contains 53 sectors, whereas the baseline economy comprises 57 sectors. This difference is due to the fact that the BEA reports sectoral data at a relatively lower degree of disaggregation for the early years of the sample.

	Dependent Varia	able: $\Delta \log Y_{s,t}$
	Data	Model
	(1)	(2)
$\Delta \log \tilde{G}_{s,t}$	$0.75^{\star\star}$ (0.30)	0.79^{***} (0.06)
$\Delta \log Y_{s,t-1}$	0.01 (0.05)	-0.01 (0.03)
$\Delta \log G_{s,t-1}$	-0.10 (0.28)	$0.03 \\ (0.06)$
Industry Fixed Effects	YES	YES
Year Fixed Effects	YES	YES
R^2	0.19	0.66
N. Obs.	2,862	3,078

Table C.5: Estimation results: Data-based versus Model-based estimates.

Notes: The table reports estimates of the coefficients in Equation (C.35) and their standard errors (between parentheses). Column (1) reports the estimates based on a panel of U.S. sectoral value added and government spending across 53 sectors, measured at an annual frequency, from 1963 to 2018. Column (2) reports the estimates based on an artificial panel of 57 sectors spanning 56 years, which is simulated using the baseline model. We closely follow Nekarda and Ramey (2011) and Acemoglu, Akcigit and Kerr (2016) by constructing the series of defense spending shocks as $\Delta \log \tilde{G}_{s,t} = \theta_s \times \Delta \log G_t$, where G_t is aggregate government spending, and θ_s is the the sample average of the ratio between sectoral defense spending and sectoral total shipments. Standard errors are double-clustered at the sector-year level.

D Sectoral Characteristics and the Aggregate Value-Added Multiplier - Model with Immobile Factors



Figure D.1: Sectoral Characteristics and the Aggregate Value-Added Multiplier.

E Aggregate Effects of Sectoral Spending Shocks - Robustness Checks

Below, we report the aggregate implications of sectoral spending shocks obtained from an alternative version of the baseline economy in which we assume that government spending does not enter directly into the utility function. This is achieved by assuming $\zeta = 1$. Figures E.1–E.3 report, respectively, the value-added, consumption, and investment multipliers associated with spending shocks in each of the 57 sectors, under this assumption. The figures show that abstracting from complementarity between private consumption and government spending in preferences reduces the magnitude of the aggregate effects of sectoral spending shocks. Intuitively, when government spending no longer raises the marginal utility of consumption, labor supply increases by a smaller amount in response to an increase in public purchases, leading to a larger crowding out of private spending, and attenuating the absolute size of the aggregate with sectoral shocks. For instance, the correlation between the aggregate value-added multipliers implied by the baseline model and the alternative economy (which abstracts from complementarity) amounts to 0.98.





Notes: The figure plots the aggregate value-added multiplier associated with each sectoral government spending shock, obtained from a version of the fully-heterogeneous economy in which government spending does not enter into the utility function.

Figure E.2: Aggregate Consumption Response to Sectoral Government Spending Shocks - No Complementarity between C_t and G_t .



Notes: The figure plots the aggregate consumption multiplier associated with each sectoral government spending shock, obtained from a version of the fully-heterogeneous economy in which government spending does not enter into the utility function.

Figure E.3: Aggregate Investment Response to Sectoral Government Spending Shocks - No Complementarity between C_t and G_t .



Notes: The figure plots the aggregate investment multiplier associated with each sectoral government spending shock, obtained from a version of the fully-heterogeneous economy in which government spending does not enter into the utility function.

F The Sectoral Composition of Government Purchases across Countries

In this section, we use the 2014 Input-Output Tables of the OECD to derive the sectoral composition of national government spending for a set of 28 countries: Austria, Belgium, Cyprus, Croatia, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Latvia, Lithuania, Luxembourg, Macedonia, the Netherlands, Norway, Poland, Portugal, Romania, Slovenia, Slovakia, Spain, Sweden, and the United Kingdom. As in Section 3.2, we consider a sequence of economies, one for each country, that feature the same steady state (based on the U.S. economy) and differ only in the calibration of the sectoral composition of the government spending shocks. Note that because the specific blend of sectoral government spending in a given country is, to a large extent, endogenous to its economic conditions, we do not seek to measure the spending multiplier in each of the 28 countries based on the model calibrated to the U.S. economy. Instead, we would like to determine what the government spending multiplier in the U.S. would be, should the calibration reflect the sectoral composition of government spending of other advanced economies.

Allowing for variation in the sectoral composition of spending of the general government across countries leads to large differences in the size of the government spending multiplier. Figures F.1 and F.2 respectively report the output and consumption multipliers associated with spending shocks whose sectoral split is calibrated to the U.S. and each of the 28 OECD countries we consider. The output multiplier ranges between 0.60 in the case of Slovakia and 0.92 for the United Kingdom. These differences are driven by the fact that the general government in (low-multiplier) countries such as Czech Republic, Slovakia, and Slovenia tends to concentrate its spending relatively more in manufacturing and transportation industries, whereas the general government of (high-multiplier) economies such as Croatia, Cyprus, and the United Kingdom spends relatively more in professional, educational, and health-care services. As for consumption, we observe – once again – strong heterogeneity, with both crowding-in and crowding-out effects associated with different countries/compositions of the public spending shock. The general picture emerging from this exercise is that the multiplier associated with the sectoral composition of government spending in the U.S. is lower than those implied by the sectoral composition of public purchases in at least half of the OECD countries.



Figure F.1: The Aggregate Value-Added Multiplier across Countries.

Notes: The figure reports the aggregate value-added multiplier based on the sectoral composition of general government spending in twenty-eight OECD countries. The red line corresponds to the value of the aggregate value-added multiplier in the United States. All the multipliers are obtained from otherwise identical economies that only differ in the sectoral composition of the government spending shock, which is calibrated based the Input-Output table of each country.





Notes: The figure reports the aggregate consumption multiplier based on the sectoral composition of general government spending in twenty-eight OECD countries. The red line corresponds to the value of the aggregate consumption multiplier in the United States. All the multipliers are obtained from otherwise identical economies that only differ in the sectoral composition of the government spending shock, which is calibrated based the Input-Output table of each country.