

Consumer Durables and Monetary Transmission in a Two-sector HANK Economy*

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Abstract

We investigate the transmission of monetary policy in a two-sector Heterogeneous Agent New Keynesian setting with both durable and nondurable consumption goods. While durable expenditure displays considerable sensitivity to the direct effects of an increase in the real rate of interest, the indirect effects associated with the general-equilibrium drop in labor demand drive the brunt of the contraction of both types of consumer spending. We highlight how agents with different access to liquid assets may denote very heterogeneous exposures to different channels of transmission, even when displaying quantitatively similar responses of their expenditure on either type of good. This represents a key input in the analysis of the effects of monetary policy, both in the aggregate and with respect to consumer inequality.

Keywords: Heterogeneous agents, durable goods, multi-sector models, monetary policy.

JEL codes: E21, E31, E40, E44, E52.

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1 Introduction

Despite making up a smaller portion of aggregate private consumption, durables play a significant role in explaining changes in private expenditures—both aggregate and at the household level (see, e.g., Stock and Watson, 1999; Attanasio, 1999)—especially in response to monetary shocks. In fact, it is widely acknowledged that consumer durable spending is much more interest-rate sensitive than nondurable spending (see, e.g., Mankiw, 1985). Due to this property, durables are commonly believed to represent an important avenue—if not the most important—for monetary policy to affect aggregate household demand (see, e.g., Erceg and Levin, 2006). Yet, much remains to be understood about the channels through which monetary shocks effectively transmit in the aggregate, through both durable and nondurable spending. On this front, a flourishing literature has emerged with the aim of incorporating rich household heterogeneity into the workhorse Representative Agent New Keynesian (RANK) model featuring one type of perishable consumption good, yielding learnings that have improved our understanding of the transmission of monetary policy. A key prediction of these Heterogeneous Agent New Keynesian (HANK) economies is that, in the presence of uninsurable income risk and some form of financial friction, general-equilibrium effects emanating from labor demand—and, in turn, affecting household disposable income—drive the brunt of the response (of nondurables) to monetary shocks (Kaplan et al., 2018; Auclert, 2019). This stands in stark contrast with the predictions of standard RANK economies, where nearly the entire response is driven by intertemporal substitution. Multi-sector RANK economies featuring durables are no exception to this property, being based on the view that durables’ interest-rate sensitivity is primarily dictated by movements in the real rate of interest (Barsky et al., 2007). Even when a collateral channel is introduced in settings featuring limited household heterogeneity (e.g., Iacoviello, 2005), general-equilibrium forces play little or no role.

Marked interest-rate sensitivity primarily arises from the demand for durables being directed towards a stock, so that even small changes in the latter can cause significant variation in the corresponding flow demand for newly produced goods. However, such a property need not exclusively hold in connection with intertemporal substitution induced by interest rate movements, if wealth and sectoral heterogeneity are significant enough to activate (and, potentially, amplify) additional channels of transmission. In this paper, we

analyze the primary channels through which monetary shocks propagate to households' expenditure on durable and nondurable goods. We do so by extending the canonical (one-sector) HANK framework to accommodate the presence of a sector that produces durable goods. Such extension appears rather important. In fact, durables are peculiar in that they have a dual functionality, being both a *consumption good*—in which case the behavior of the user cost, as primarily shaped by movements in the relative price and the real rate of interest, is key to expenditure allocation on either good—and as a *store of value* that can be accumulated and traded on second-hand markets—so as to transfer wealth across time. Both these properties characterize standard multi-sector RANK models with durables. However, the way they combine to dictate the response of durable spending—and, in turn, of aggregate expenditure—to monetary policy shocks is likely to be shaken by the introduction of sizable uninsurable idiosyncratic risk, in which case the conditional behavior of households' disposable income may represent a viable channel for sectoral transmission.

Our HANK model retains the building blocks of standard two-sector New Keynesian models with asymmetric price stickiness between sectors, in the vein of Barsky et al. (2007) and Monacelli (2009), augmented to reflect uninsurable idiosyncratic risk on the household side, with some households being constrained in the access to liquid assets.¹ We use this framework as a laboratory to decompose the response of consumption on both durables and nondurables to a monetary policy shock into a *direct* (or interest-rate) effect—as mostly dictated by intertemporal substitution—and an *indirect* effect, which operates through general-equilibrium changes in households' disposable income. In turn, the latter is further decomposed into the response that can be reconducted to changes in the *relative price* of durables to nondurables, and *pure income* effects.

The main result we report is that not only the response of nondurables is mostly driven by pure income effects—in line with the quantitative insights from one-sector HANK economies à la Kaplan et al. (2018)—but also that of durables is predominantly affected by general-equilibrium forces. Yet, unlike nondurables, durables do display sizable interest-rate effects that, in turn, have grip on aggregate consumption. We then show how pure income effects are key to overcoming the relative-price force that induces consumers to substitute durables for nondurables and *vice-versa*—depending on the relative degree of

¹Relative to the HA literature that deals with durable expenditure at the household level, we focus on durable adjustment along the intensive margin, rather than on the extensive margin (in this respect, see Berger and Vavra, 2015, Harmenberg and Öberg, 2021, McKay and Wieland, 2021, among others).

sectoral price stickiness and the sign of the shock—thus resolving the comovement puzzle that typically plagues otherwise standard two-sector RANK models with asymmetric sectoral price stickiness (see, e.g., Barsky et al., 2007). In this respect, decomposing consumer expenditure responses based on the holdings of liquid assets highlights some distinctive traits of different household types, and how they react to monetary shocks. As for the savers, the sharp reallocation of resources between their stock of durables and bond holdings in the face of a monetary contraction reflects sizable capacity of the interest-rate channel. As for liquidity-constrained households, instead, they consistently display positive conditional comovement in consumption—with durables denoting much stronger reactivity—thus making these agents’ consumption habits and share in the total population decisive for resolving the comovement puzzle.

These results are robust to realistic extensions to the baseline framework, such as deficit financing. We also augment the model to feature sticky wages (see, e.g., Auclert et al., 2020b). In this case, pure income effects are responsible for an even larger portion of the response of expenditure on both types of goods—being relative-price effects relatively muted—while direct effects are much less of a driver of the response of durables (and, in turn, of aggregate consumption), as compared with the case of flexible wages.

All in all, our results are important in that they should lead us to rethink the most effective channels of monetary-policy transmission in HA economies, when focusing on both aggregate consumption and its components characterized by different degrees of durability and price stickiness. Moreover, we convey useful insights about the exposure of households with heterogeneous financial access to different channels of transmission, a fact that the existing literature has mostly emphasized in connection with nondurables. In fact, even with respect to durable spending, it is the case that liquidity-constrained households mostly respond to pure income effects, while savers’ reaction is predominantly shaped by intertemporal substitution (as predicted by most of the RA literature). Evaluating the implications of this property for the conditional behavior of consumption and income inequality may represent an important research avenue.

Related literature We relate to a burgeoning literature on monetary policy transmission in New Keynesian models with rich wealth distributions. Our work is inspired by the seminal work of Kaplan et al. (2018), who investigate the effects of monetary policy in a rich calibrated one-sector HANK model (see also Alves et al., 2020). Another relevant con-

tribution is Auclert (2019), who reports that redistribution triggered by monetary policy is key in amplifying its effects in the aggregate, and shows how accounting for durables' interest-rate sensitivity may be important in quantifying the redistribution elasticity of total consumption to the real interest rate. We also relate to McKay and Wieland (2022), who build a model that features durable adjustment along the extensive margin, exploiting its sensitivity to the (contemporaneous) user cost to address the forward guidance puzzle. We abstract from this channel, while casting an otherwise standard two-sector NK model in a HA setting, so as to retain closer comparability with a long-standing tradition of studies that examine monetary transmission in multi-sector economies.

Our paper also relates to a large literature tackling the *comovement puzzle* that typically characterizes standard two-sector New Keynesian models with asymmetric price rigidity. Remedies that have been put forward to address this puzzle can essentially be divided in three categories: *i*) opting for non-separable preferences between a composite of sectoral consumption goods and labor supply (see, e.g., Dey and Tsai, 2012; Katayama and Kim, 2013); *ii*) adopting sticky prices of the production inputs, such as Carlstrom and Fuerst (2010)—who assume sticky wages—or Sudo (2012) and Petrella et al. (2019), who both allow for input-output interactions; *iii*) embedding financial frictions in the vein of Tsai (2016)—who stresses the role of working capital—or Monacelli (2009), who emphasizes the importance of the collateral constraints applying to households. All of these modeling devices influence the drop in the relative price of durables (the latter are typically assumed to display prices that are more flexible than those of nondurables), in the face of a monetary tightening. Our framework takes a different route, and reproduces sectoral comovement not by impairing the relative-price channel, but by highlighting the importance of transitory income movements in the presence of market incompleteness.

Structure The paper is structured as follows: Section 2 details the baseline two-sector HANK model. Section 3 details the calibration and the solution of the deterministic steady state. In Section 4 we perform various decompositions of aggregate and household-level responses to a monetary tightening, and test the robustness of our results to extending the baseline model so as to account for deficit financing and sticky wages. Section 5 concludes.

2 A two-sector HANK model with durables

The economy is populated by households with preferences over durable and non-durable goods, as well as labor hours that are supplied to intermediate-goods firms operating in a regime of monopolistic competition. Households are subject to idiosyncratic productivity shocks, and face a borrowing constraint. Intermediate-goods firms sell their products to firms operating in a perfectly-competitive final-goods sector. The government pursues monetary policy, while balancing its budget on a period-by-period basis. The remainder of this section details the key blocks of the model, as well as how equilibrium obtains.

2.1 Households

We assume a continuum of households, indexed by $s \in [0, 1]$. Consumer preferences are defined over (a Cobb-Douglas aggregator of) nondurable consumption and the stock of durables— $C_{n,t}(s)$ and $D_t(s)$, respectively—² as well as over labor hours, $\mathcal{N}_t(s)$. Households' intertemporal utility reads as

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{n,t}^\theta(s) D_t^{1-\theta}(s))^{1-\sigma}}{1-\sigma} - \psi_N \frac{\mathcal{N}_t^{1+\varphi}(s)}{1+\varphi} \right] \right\}. \quad (1)$$

We define the durable flow as $C_{d,t}(s) = D_{t+1}(s) - (1 - \delta)D_t(s)$. Household s 's budget constraint (deflated by the price of nondurables) is given by

$$C_{n,t}(s) + Q_t C_{d,t}(s) + B_{t+1}(s) = (1 + r(B_t(s)))_t B_t(s) + w_{n,t} N_t \exp \{e_t(s)\} + Div_t \bar{Div}(s) - \tau_t \bar{\tau}(s) - \frac{\alpha}{2} \left(\frac{C_{d,t}(s)}{D_t(s)} \right)^2 D_t(s), \quad (2)$$

²Concerning the implications for the transmission of monetary impulses through movements in the relative price, the assumption of Cobb-Douglas preferences is rather conservative, as the empirical estimates of the substitution elasticities between durables and nondurables range from below to around one; see Ogaki and Reinhart (1998), Davis and Ortalo-Magné (2011), Pakos (2011) and Albouy et al. (2016). We stand at the high end of the range of these estimates.

where $B_{t+1}(s)$ denotes bond holdings, Q_t is the price of durables relative to that of non-durables, $w_{n,t}$ is the real wage rate,³ α scales the adjustment cost on durables, $\delta \in [0, 1]$ is the depreciation rate and $e_t(s)$ is an idiosyncratic productivity shock with unit mean. Furthermore, $r(B_t(s))_t$ is the real return on bonds when $B_t(s) > 0$, while it equals the real rate plus a borrowing wedge, κ , when $B_t(s) < 0$ (see Kaplan et al., 2018). Households pay taxes, τ_t , and receive dividends from the ownership of firms, Div_t , according to the incidence rules $\bar{\tau}(s)$ and $\bar{Div}(s)$, which are set so that taxes and dividends are linear functions of individual productivity. Finally, households face a borrowing constraint:

$$B_t(s) \geq -\psi Y, \quad (3)$$

where Y is steady-state total output, and ψ is a scaling parameter. We assume that all households supply labor according to the solution in the RA representation of the model (see, e.g., Debortoli and Galí, 2021) under perfect labor mobility between sectors, that is:

$$w_{n,t} = \psi N_t N_t^\varphi \frac{1}{\theta} (C_{n,t}^\theta D_t^{1-\theta})^\sigma \left(\frac{C_{n,t}}{D_t} \right)^{1-\theta}, \quad (4)$$

where $C_{n,t} \equiv \int_0^1 C_{n,t}(s) ds$ and $N_t = \mathcal{N}_t(s)$ for all s .⁴

2.2 Production

Final-goods producers There are two sectors, indexed by $j = \{n, d\}$. Two representative sectoral final-goods producers aggregate a continuum of intermediate goods indexed by $i \in [0, 1]$, $y_{j,t}(i)$ (with price $p_{j,t}(i)$), in accordance with the CES technology

$$Y_{j,t} = \left(\int_0^1 y(i)_{jt}^{\frac{\epsilon_j-1}{\epsilon_j}} di \right)^{\frac{\epsilon_j}{\epsilon_j-1}}, \quad (5)$$

³Formally, this is indexed by “ n ”, as we deflate the nominal wage by the price level of nondurables. However, it is important to recall that, as we assume perfect labor mobility, nominal wages are equalized across sectors.

⁴Taking a representative-agent stand on labor supply allows us to dampen wealth effects for low liquidity households, which helps reconcile the model with the available evidence (see, e.g., Auclert et al., 2020a).

where ϵ_j is the elasticity of substitution across goods of type j . Given $Y_{j,t}$, profit maximization for the j th final goods producer implies a demand for intermediate good i in the same sector:

$$y(i)_{j,t} = y(p(i)_{j,t}; P_{j,t}, Y_{j,t}) = \left(\frac{p(i)_{j,t}}{P_{j,t}} \right)^{-\epsilon} Y_{j,t}, \quad (6)$$

where $P_{j,t}$ denotes the equilibrium price of the final good:

$$P_{j,t} = \left(\int_0^1 p(i)_{j,t}^{1-\epsilon_j} di \right)^{\frac{1}{1-\epsilon_j}}. \quad (7)$$

Intermediate-goods producers Intermediate-goods producers in either sector employ a linear production technology:

$$Y_{j,t}(i) = A_j N_{j,t}(i), \quad (8)$$

where A_j represents total factor productivity, assumed to be common to all firms in sector j . Price setting in each sector is subject to virtual Rotemberg adjustment costs $\mathcal{C}_j(\cdot) = \frac{\xi_j}{2} \left(\frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t}$ (with $\xi_j > 0$) as in, e.g., Hagedorn et al. (2019). Each firm's value function in real terms reads as

$$V_{j,t}(p(i)_{j,t-1}) \equiv \max_{p(i)_{j,t}} \frac{p(i)_{j,t}}{P_{j,t}} y(p(i)_{j,t}; P_{j,t}, Y_{j,t}) - w_{j,t} N_{j,t} - \frac{\xi_j}{2} \left(\frac{p(i)_{j,t}}{p(i)_{j,t-1}} - 1 \right)^2 Y_{j,t} + \beta V_{j,t+1}(p(i)_{j,t}). \quad (9)$$

This problem yields the usual New Keynesian Phillips curve(s):

$$(1 - \epsilon_j) + \epsilon_j w_{j,t} / A_j - \xi_j (\Pi_{j,t} - 1) \Pi_{j,t} + \beta \xi_j (\Pi_{j,t+1} - 1) \Pi_{j,t+1} \frac{Y_{j,t+1}}{Y_{j,t}} = 0, \quad (10)$$

while total real dividends (deflated by $P_{n,t}$) are

$$Div_t = \sum_j Div_{j,t} = Y_{n,t} - w_{n,t}N_{n,t} + Q_t (Y_{d,t} - w_{d,t}N_{d,t}). \quad (11)$$

2.3 Policy

Monetary policy Monetary policy sets the nominal rate according to a Taylor rule that features a non-systematic component, u_t^r :

$$i_t = \phi_{\tilde{\pi}} \tilde{\pi}_t + u_t^r, \quad (12)$$

where $\tilde{\pi}$ is the net (aggregate) rate of inflation, with $\tilde{\Pi}_t \equiv \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^\gamma$, $\gamma \in [0, 1]$.

Fiscal policy The fiscal authority issues one-period nominal bonds, B^g , maintaining this constant in fulfillment of the steady-state bond-to-output ratio, and adjusts the level of lump-sum taxes, τ_t , to balance its budget period-by-period:

$$\tau_t = r_t B^g. \quad (13)$$

2.4 Equilibrium

Market clearing Bonds market clearing obtains as

$$B_t = \int_0^1 B_t(s) ds = B^g. \quad (14)$$

Aggregate labor hours are given by

$$N_t = \sum_j \int_0^1 N_{j,t}(i) di = \sum_j Y_{j,t}/A_j, \quad (15)$$

and are assumed to be distributed uniformly among household types, i.e. $N_t(s) = N_t$ for all $s \in (0, 1)$. The sectoral resource constraints are

$$Y_{d,t} = C_{d,t}, \quad (16)$$

and

$$Y_{n,t} = C_{n,t} + \chi_t + \kappa \int \max(-B_t(s), 0) ds, \quad (17)$$

where the last two terms of (17) respectively capture the costs of adjusting the stock of durables and that of borrowing, respectively. It follows from equations (16) and (17) that the market for aggregate goods clears in accordance with

$$Y_t = Q_t Y_{d,t} + Y_{n,t} = Q_t C_{d,t} + C_{n,t} + \chi_t + \kappa \int \max(-B_t(s), 0) ds. \quad (18)$$

Equilibrium definition An equilibrium in this economy is defined as paths for individual household decisions, $\{C_{n,t}(s), D_t(s), B_t(s)\}_{t \geq 0}$, inflation rates and relative prices, $\{\Pi_{n,t}, \Pi_{d,t}, Q_t\}_{t \geq 0}$, real wages, $\{w_{n,t}, w_{d,t}\}_{t \geq 0}$, sectoral output and employment, $\{Y_{n,t}, Y_{d,t}, N_{n,t}, N_{d,t}\}_{t \geq 0}$, dividends, $\{Div_t\}_{t \geq 0}$, interest rates, $\{i_t, r_t\}_{t \geq 0}$, government bond supply and taxes, $\{B_t^g, \tau_t\}_{t \geq 0}$, such that:

1. Households maximize their objective functions, given the $\{Q_t, r_t, w_{n,t}, N_t, Div_t, \tau_t, \}_{t \geq 0}$ sequences;
2. Firms in each sector maximize their profits, taking as given the $\{w_{n,t}, w_{d,t}\}_{t \geq 0}$ sequences;
3. Given the $\{C_{n,t}, D_t\}_{t \geq 0}$ sequences, the real-wage sequences, $\{w_{n,t}\}_{t \geq 0}$ and $\{w_{d,t}\}_{t \geq 0}$ are consistent with the wage schedule, (4), conditional on perfect sectoral mobility, as captured by $Q_t w_{d,t} = w_{n,t}$;
4. The government budget constraint, (13), is satisfied;
5. Bonds, labor, nondurable and durable goods markets clear;
6. Distributions fulfill consistency requirements.

3 Calibration

An overview of our calibration is reported in Table 1. Each period in the model corresponds to a quarter. We calibrate the discount factor, β , so the steady-state annual real risk-free rate is 3 percent. The coefficient of relative risk aversion, σ , and the inverse Frisch elasticity of labor supply, φ , are set to 1. The utility weight on nondurables, θ , is set to 0.7, so as to match the 60 percent steady-state ratio of nondurables to total consumption; a value in the middle of the range provided in Beraja and Wolf (2021). Durables' depreciation, δ , is set to 0.068, as in McKay and Wieland (2021). The idiosyncratic income parameters, σ_e and ρ_e , are set to 0.1928 and 0.9777, respectively, following McKay et al. (2016) and Auclert (2019). On the supply side, we set ϵ_n and ϵ_d to 6, as in Monacelli (2009). As for the policy parameters, the steady-state government debt-to-output ratio is set to 0.26, as in Kaplan et al. (2018). The reaction parameter in the Taylor rule, ϕ_π , is set to 1.5. The weight on durables in the monetary authority's inflation index, γ , is set to the steady-state share of durable consumption to total consumption, 0.4. Finally, we implement the simulated method of moments (SMM), using α and ξ_n, ξ_d to target: *i*) the relative volatility of durable to nondurable expenditure, calculated using HP-filtered log-data;⁵ *ii*) the stickiness of durable and nondurable prices. As for the latter, Nakamura and Steinsson (2008) report a median price duration of 8-11 months (with one of the most prominent types of durable, *transportation goods*, exhibiting a price duration of 2.7 months; see their Table II). We target Calvo probabilities $\theta_n^{Calvo} = 0.75$ and $\theta_d^{Calvo} = 0.25$, thus imposing durables to be more price-flexible than nondurables (see, e.g., Klenow and Malin, 2010).⁶

We note that, based on this calibration, the unconditional correlation between durable and nondurable expenditure amounts to 0.495 (conditional on our baseline monetary policy shock, and measured over 10 quarters), which is very close to the analogous computed with NIPA HP-filtered data (0.422).

⁵Relative volatility is computed from *on-impact* responses to a 0.25% innovation to the non-systematic component of the Taylor rule, assuming this is an AR(1) process with an autoregressive coefficient equal to 0.5, as in Kaplan et al. (2018).

⁶The coefficient θ_j^{Calvo} is defined as the probability for a firm in sector j of not being able to adjust prices in a given quarter. From our calibration exercise, we obtain $\theta_n^{Calvo} = 0.62$ and $\theta_d^{Calvo} = 0.40$ —corresponding to median price durations of 7 and 5 months, respectively—and a on-impact relative volatility of 3.560. This value is in line with the evidence of Erceg and Levin (2006) and Sterk and Tenreyro (2018), among others. To determine sector-specific Rotemberg adjustment costs, we rely on their mapping with the Calvo probabilities, as implied by $\xi_j = \theta_j^{Calvo} (\epsilon_j - 1) / ((1 - \theta_j^{Calvo})(1 - \beta\theta_j^{Calvo}))$.

Table 1: Baseline model calibration

Parameter	Value	Target/Source
Household parameters		
β	0.9652	Steady-state adjustment
σ	1	Std. business-cycle literature value
φ	1	Std. business-cycle literature value
θ	0.7	$\frac{C_n}{C_n+C_d}$; Beraja and Wolf (2021)
α	0.119	SMM target volatility of $C_d/C_n = 3.572$; BEA, NIPA accounts
δ	0.068	BEA Fixed Asset, McKay and Wieland (2021)
ψ_N	0.764	Steady-state adjustment
ψ	0.833	Borrowing limit based on earnings
κ	0.0465	Steady-state share of households with $B(s) = 0$; Kaplan et al. (2018)
ρ_e	0.9777	McKay et al. (2016) and Auclert (2019)
σ_e	0.1928	McKay et al. (2016) and Auclert (2019)
Supply-side paramaters		
r	0.03/4	Debortoli and Galí (2021)
ϵ_n, ϵ_d	6	Monacelli (2009)
ξ_n	20.21	SMM target Calvo probability of 0.75; Nakamura and Steinsson (2008)
ξ_d	5.43	SMM target Calvo probability of 0.25; Nakamura and Steinsson (2008)
A_n	1.0	Steady-state adjustment
A_d	2.16	Steady-state adjustment
Policy parameters		
B^g/Y	0.26	Liquid assets/GDP; Kaplan et al. (2018)
ϕ_π	1.5	Taylor (1993)
γ	0.40	$C_d/(C_n + C_d)$

3.1 Deterministic steady state

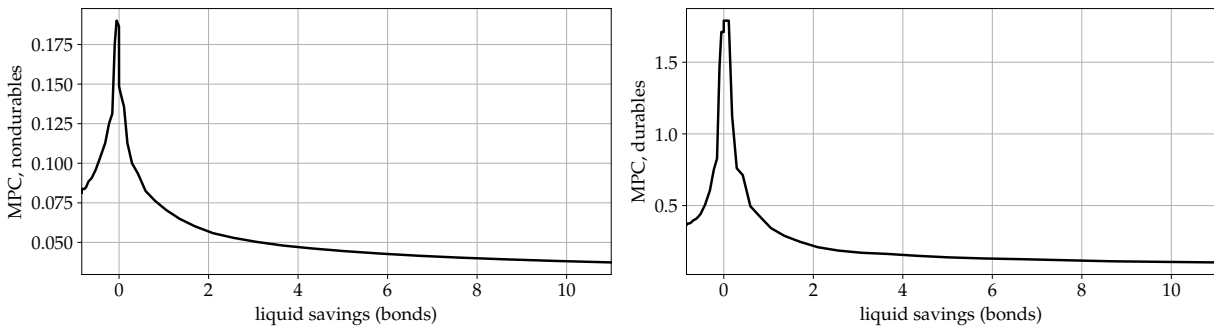
Let a generic variable x_t be denoted by x in the steady state. When solving for the steady state, we use a multi-dimensional root finder to guess on β, Q, N_d and target: *i*) bonds market clearing; *ii*) durable goods market clearing; *iii*) total employment ($N = 1$). Given bonds and durable goods markets clearing, the nondurable goods market clears by Walras' law. The household solution is obtained using the endogenous grid method algorithm (EGM) of Auclert et al. (2021) in two dimensions; see Appendix A for details. The steady-state household distribution of is retrieved by relying on the deterministic histogram method of Young (2010). Given guesses for β, Q, N_d , we can solve for equilibrium quantities, both at the aggregate and at the household level, as described in Appendix B.

We obtain a steady-state skewness of the durable stock over nondurable consumption of 0.867, which is remarkably in line with microeconomic evidence in Bertola et al. (2005) (see Figure 8 in Appendix F for a density plot), especially if we consider that the present

framework does not feature any adjustment along the extensive margin. In addition, Figure 1 contains two panels reporting the steady-state marginal propensity to consume (MPC) nondurables and durables, respectively, both as a function of the holdings of liquid assets. Both MPCs roughly peak at the point where bond holdings are nil due to the debt cost, as captured by the borrowing wedge, κ . Notice that households with zero liquidity but median holdings of durables can use the durable stock as a self-insurance device (yet, subject to an adjustment cost). As such, durables assume the dual role of a consumption good and of a relatively illiquid asset, at the eyes of “wealthy hand-to-mouth” households (see Kaplan et al., 2018). Despite this feature, the MPCs are still relatively large for households who are constrained in the access to liquid savings.

At the aggregate level, the model features average marginal propensities to consume (MPC) and spend (MPX) that are empirically realistic. In quarterly terms, the MPC is 12.8% for nondurables, while the MPX for durables amounts to 138.4%. As for total expenditure, the MPX amounts to 76.9%. Thus, the MPC for nondurables is slightly below the empirical estimates of about 15-25% for nondurables, while the total MPX is well within the 50-90% range of the available estimates (Laibson et al., 2022).

Figure 1: Marginal propensity to consume as a function of liquid savings



Note: To plot the MPCs we fix the idiosyncratic income shock, $\epsilon(s)$, as well as the stock of durables, $D(s)$, at their median steady-state values.

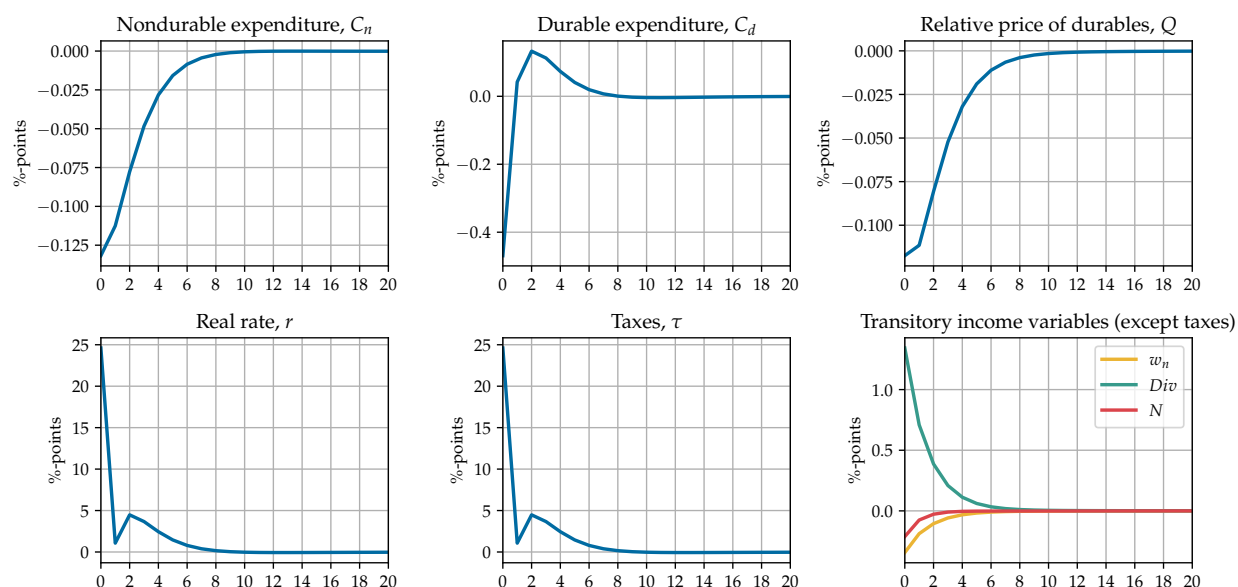
4 Monetary transmission

We are now ready to study monetary transmission, with a special focus on how the response of different types of expenditure, both at the aggregate and at the household level, can be decomposed into direct and (different) indirect effects. We then test the robustness

of our main insights by accounting for realistic extensions to the original framework.

4.1 Impulse responses to a monetary policy shock

Figure 2: Impulse responses to a contractionary monetary policy shock



Note: We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

We consider a monetary policy shock at time $t = 0$. As in Kaplan et al. (2018), we take a quarterly innovation of 0.25%, while the shock-persistence parameter is set to 0.5. To obtain general-equilibrium impulse responses, we solve the model by approximating it to the first order, around the deterministic steady state, using the sequence-space method described in Auclert et al. (2021).⁷ The results of the experiment are presented in Figure 2. We may readily notice how the monetary shock pushes both expenditures down, with the durable one featuring a relatively deeper drop, followed by a hump-shaped recovery, as it has typically been shown in both theoretical and empirical settings (see, e.g., Beraja and Wolf, 2021). Also the drop in the relative price is consistent with what expected on *a priori* grounds, given that durables feature relatively more flexible prices. Nevertheless, the magnitude of the relative price change is relatively modest, as reported by McKay and Wieland (2021), among others. The main scope of the subsequent analysis is to study

⁷For the sequence-space formulation of the model, we refer the reader to Appendix C.

the determinants of the contraction in both types of expenditure, as well as their relative strength.

4.2 Response decomposition

Following Kaplan et al. (2018), we decompose the response of different expenditures as of $t = 0$ into *direct* (i.e., interest-rate) and *indirect* (i.e., general-equilibrium or transitory income) effects, by total differentiation of the impulse-response path of $\{C_{j,t}\}_{t \geq 0}$, for $j = \{n, d\}$:

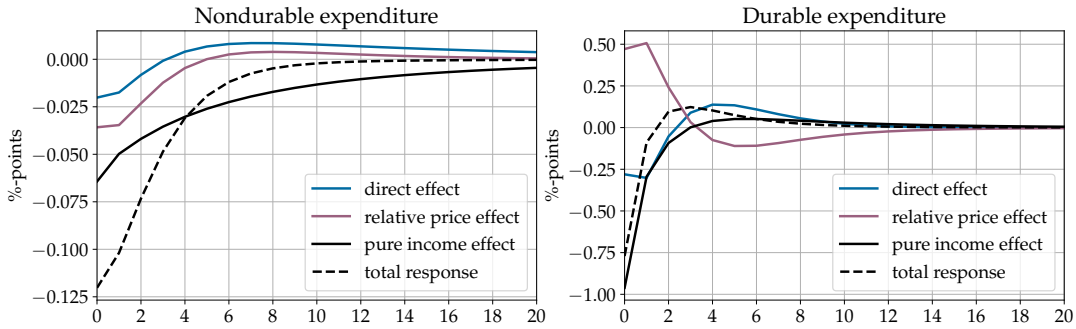
$$dC_{j,0} = \underbrace{\sum_{t=0}^{\infty} \frac{\partial C_{j,0}}{\partial r_t} dr_t}_{\text{direct effect}} + \underbrace{\sum_{t=0}^{\infty} \left(\underbrace{\frac{\partial C_{j,0}}{\partial Q_t} dQ_t}_{\text{relative-price effect}} + \underbrace{\frac{\partial C_{j,0}}{\partial N_t} dN_t + \frac{\partial C_{j,0}}{\partial w_{n,t}} dw_{n,t} + \frac{\partial C_{j,0}}{\partial Div_t} dDiv_t + \frac{\partial C_{j,0}}{\partial \tau_t} d\tau_t}_{\text{pure income effects}} \right)}_{\text{indirect effects}}. \quad (19)$$

Each effect is computed by moving only the variable with respect to which the partial differential is taken. For example, the direct effect is a partial-equilibrium one, whereby all variables other than the real rate are kept fixed. As we are in a two-sector setting, indirect effects can be further grouped into a *relative-price* effect—which embodies both income and substitution effects—and terms that exclusively exert *pure income* effects. Numerically, we calculate the partial-equilibrium household paths by varying only the relevant inputs, while keeping the remaining terms fixed. For example, in the case of the direct effect on nondurable consumption, we compute

$$\sum_{t=0}^{\infty} \frac{\partial C_{n,0}}{\partial r_t} dr_t = \sum_{t=0}^{\infty} \left(\int \frac{\partial C_{n,0}(e_t(s), B_t(s), D_t(s); \{r_t, Q, w_n, N, Div, \tau\}_{t > 0})}{\partial r_t} ds \right) dr_t. \quad (20)$$

In practice, this is accomplished by varying one input at a time, given the general-equilibrium path computed through household Jacobians, which are calculated when tackling the sequence-space solution of the impulse-response functions.

Figure 3: Expenditure response decomposition



Note: Decomposition of the response of nondurable and durable expenditure into direct, relative-price and pure income effects. We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

Figure 3 reports our baseline expenditure response decomposition. This shows how direct and pure income effects push both durables and nondurables down. By contrast, the fall in the relative price would *per se* lead to substitute nondurables for durables, potentially yielding an empirically counterfactual negative comovement. In fact, summing the relative-price to the direct effect would still imply negative comovement between durables and nondurables, as the intratemporal substitution motive—which is driven by the drop in Q_t —is way more powerful than intertemporal substitution, as is typically the case in standard two-sector RANK models with sticky prices. Thus, pure income effects prove key to generating positive consumption comovement.

From a quantitative viewpoint, the on-impact interest-rate effect amounts to -0.025 percentage points (pp) for nondurables, while pure income and relative-price effects amount to -0.056 pp and -0.050 pp, respectively. As for durables, the corresponding figures are -0.37 pp, -0.80 pp and 0.70 pp, respectively. Over a year, the contribution of the direct effect is 15% for nondurables and 374% for durables, while the contribution of pure income effects amounts to 47% and 647% for nondurables and durables, respectively.⁸ All in all, the contribution of income-related effects to the fall in both types of expenditure is roughly twice as large as that of the direct effect. Looking at the effect of the relative price in isolation, instead, we measure a contribution of 38% and -921% for nondurables and durables, respectively.

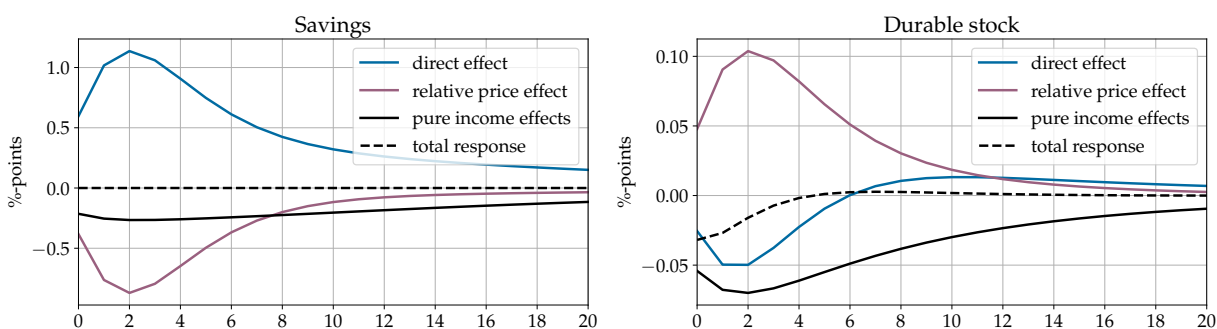
Notably, direct, pure income and relative-price effects respectively contribute by 49%,

⁸To establish a term of comparison, in Kaplan et al. (2018) the relative contribution of the direct effect to the response of (nondurable) consumption amounts to about 20% over the year after the shock.

99% and -48% of the response of aggregate expenditure, thus indicating a roughly even contribution of direct and total indirect effects in the baseline two-sector HANK. This implies that monetary transmission through intertemporal substitution has some grip, relative to what indicated by one-sector HANK models with nondurables only.

We can further decompose indirect effects into a variety of sub-components. We do this in Appendix F, Figure 9. Here, we see that the brunt of the negative effect from the income components arises from labor-related variables (N and w_n). Taxes account for a smaller share of the total negative push. This can be explained upon taxes being imposed based on productivity, so that low-income households—who are more sensitive to transitory income shocks—are partially insulated from fiscal propagation. From Kaplan et al. (2018), it is well known that the exact assumptions about how the government budget constraint adjusts outside the steady state matter when budgets are balanced period-by-period. In Section 4.3 we show that our results still hold in the presence of deficit financing. Moreover, one should recall that dividends are expansionary in the present scenario, as is typically the case in New Keynesian economies featuring rigid prices. In light of this, we argue that “positive-comovement” forces would be even stronger in a similar model where dividends are procyclical. To test such conjecture, Section 4.4 introduces sticky wages.

Figure 4: Portfolio-based response decomposition



Note: Decomposition of the response of liquid savings and the durable stock into direct, relative-price and pure income effects. We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

A portfolio-based decomposition We should elaborate further on the specifics and the implications of durables displaying marked interest-rate sensitivity. A useful perspective to examine this issue consists of considering that, being a store of value, durables are

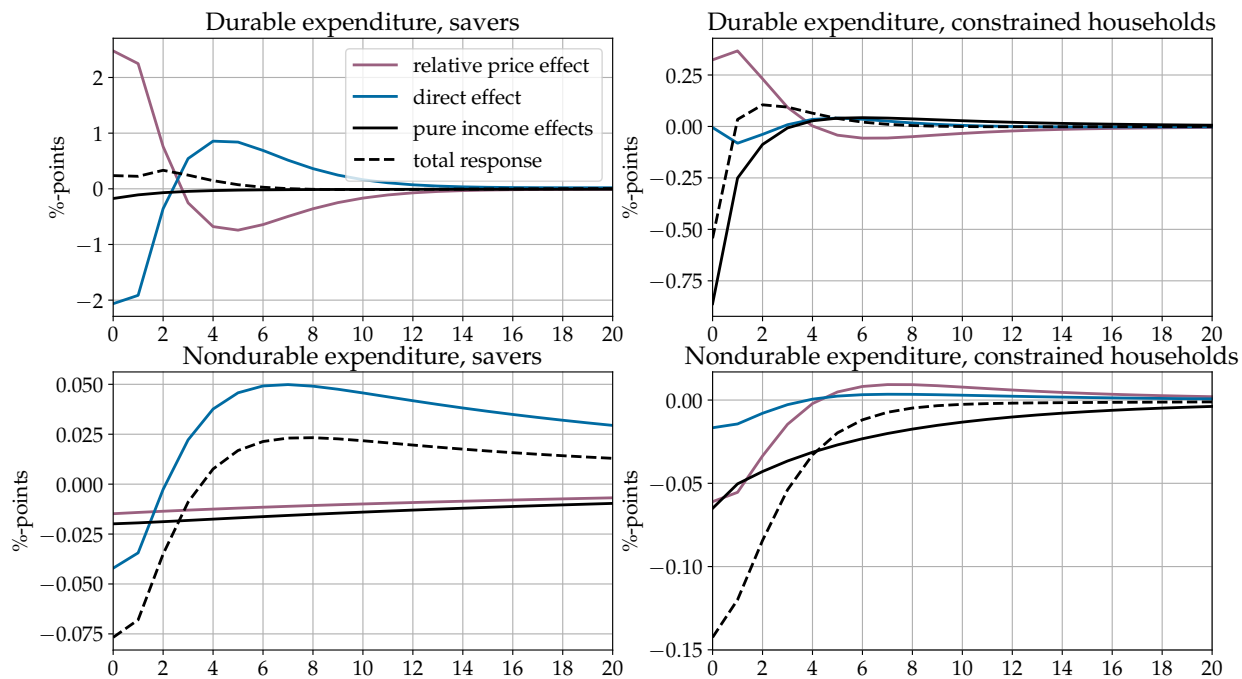
implicitly involved in a portfolio allocation choice, together with liquid financial assets (whenever agents have access to financial investment). Therefore, we consider a response decomposition of the economy-wide *portfolio* featuring bonds and the stock of durables (see Figure 4). Focusing first on the direct effects of the monetary tightening, we report a conditional negative comovement between the holdings of the two assets, with bonds denoting much stronger reactivity than durables. As the return on liquid assets increases, households who are not constrained in the access to financial investment are progressively induced to tilt their portfolio towards bonds. On the other hand, intertemporal substitution has limited grip on the response of the stock of durables, both because adjustment is frictional—applying to all households, based on their durable holdings and investment—and because liquidity constrained consumers are predominantly affected by pure income effects, as we will soon see in detail. As for the force emanating from the contraction in the relative price, this would *per se* induce households to increase their holdings of durables, while reducing bond holdings. Pure income effects, instead, are inevitably contractionary with respect to both types of assets, being substantially stronger for bonds. All in all, the sum of these effects is nil for liquid assets—by virtue of market clearing and a fixed supply—whereas the stock of durables contracts, with pure income effects being primarily responsible for this. Based on this evidence, the next step in the analysis consists of understanding to which extent the reaction of durable expenditure changes depending on households’ holdings of liquid assets, and whether different channels assume different importance with respect to this liquidity-based household-sorting criterion.

A decomposition based on the holdings of liquid assets The first row of Figure 5 breaks down the response of durable expenditure, both for liquidity-constrained households (i.e., households with zero or negative bond holdings), and for savers (i.e., households with positive bond holdings).⁹ Confirming the portfolio-based analysis, we see that savers’ durable expenditure is very interest-rate sensitive, given the motive to re-balance the portfolio of assets to move away from durables and towards bonds. In spite of this, changes in the relative price are the main driver of their durable expenditure, to the extent that joint contractionary force exerted by intertemporal substitution and other income effects is overcome. As for liquidity-constrained households, instead, the contraction in

⁹We opt for a standard sorting of households into liquidity constrained and savers, based on the holdings of liquid assets (see, e.g., Kaplan et al., 2014, 2018).

the relative price has a relatively muted impact on their durable expenditure. Analogously, limited reactivity to the interest-rate effect is displayed—as expected in light of the hand-to-mouth behavior characterizing this class of consumers—while most of the hit is taken through pure income effects. Shifting the focus to nondurable expenditure, all forces are contractionary, with the peculiarity that pure income and relative-price effects are very similar, at least on impact, for both types of consumers. The overall picture emerging from these “household-level” decompositions is that negative comovement between durable and nondurable expenditure appears as a distinctive trait of savers’ consumption response in the face of a monetary disturbance, at least in a model featuring flexible wages and asymmetric sectoral price stickiness.¹⁰ By contrast, constrained agents display positive conditional comovement through marked pure income effects, a property that renders these agents and their consumption habits decisive for resolving the comovement puzzle in the HANK model with asymmetrically sticky sectoral prices.

Figure 5: Expenditure response decomposition by steady-state bond holdings



Note: Decomposition of nondurable and durable expenditure responses into direct, relative-price and pure income effects, for households differing with respect to their steady-state holdings of liquid assets (bonds). Liquidity-constrained households are defined as households with zero or less liquid assets. Savers are defined as households holding above zero liquid assets. The effects are calculated by subtracting either expenditure—conditional on the holdings of assets on either side of a the ergodic distribution with respect to the truncation rule—from the shock-response counterpart. We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

¹⁰In Section 4.4 we show how savers also display positive comovement, in light of sticky wages limiting movements in the relative price, so that also their durable expenditure contracts.

4.3 Deficit financing

It is well known that the specific assumptions about how the government budget constraint adjusts outside the steady state matter in HANK economies, especially when governments balance their budget period-by-period. As mentioned in Section 4.2, part of the positive consumption comovement accomplished through pure income effects is driven by the tax increase. Thus, to test the robustness of this result, we neutralize movements in taxes by replacing equation (13) with (21), as in Auclert et al. (2020b):

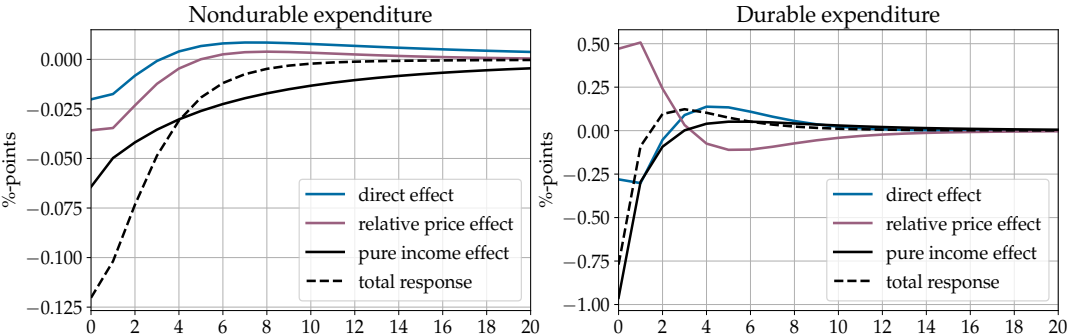
$$(1 + r_t) B_{t-1}^g = \tau_t + B_t^g, \tag{21}$$
$$\tau_t = \tau + \phi_\tau (B_{t-1}^g - B^g),$$

where τ and B^g denote steady-state taxes and government bonds, respectively, while ϕ_τ determines how fast deficits are closed. Note that such formulation does not affect the steady state. Outside the steady state, we determine taxes in each period conditional on the government budget constraint holding; see Appendix D for further details.

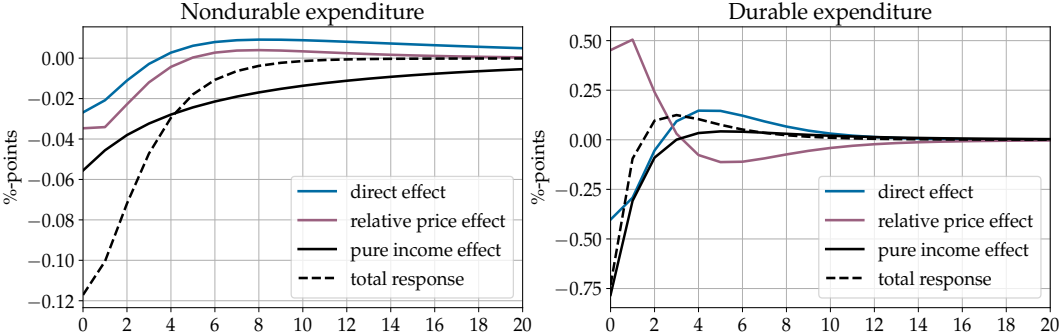
Re-calibration We set ϕ_τ to 0.1, as in Auclert et al. (2020b). Note that, under deficit financing, we need to re-perform our SMM calibration exercise to determine a value of the scaling parameters in the adjustment cost of durables and the price-adjustment costs, while targeting the volatility of durables to nondurables. Doing so results in $\alpha = 0.137$, while the Calvo probability amounts to 0.62 for nondurables and 0.40 for durables, thus mapping into $\xi_n = 19.90$ and $\xi_d = 8.37$, respectively. The discount factor, β , is now 0.965, the borrowing wedge, κ , is 0.0454, while the total factor productivities for nondurable and durable production are 1.0 and 2.15, respectively. Finally, the scaling parameter for labor disutility, ψ_N , is 0.765. The resulting volatility of durables-to-nondurables is 3.563, while the steady-state ratio between nondurable and total consumption is 0.60 (implying $\gamma=0.4$), in line with the baseline calibration.

Decomposition of consumption responses The second row of Figure 6 contains a consumption decomposition of the effects induced by a monetary tightening in this model extension, in line with the analogous decomposition for the baseline model in Section

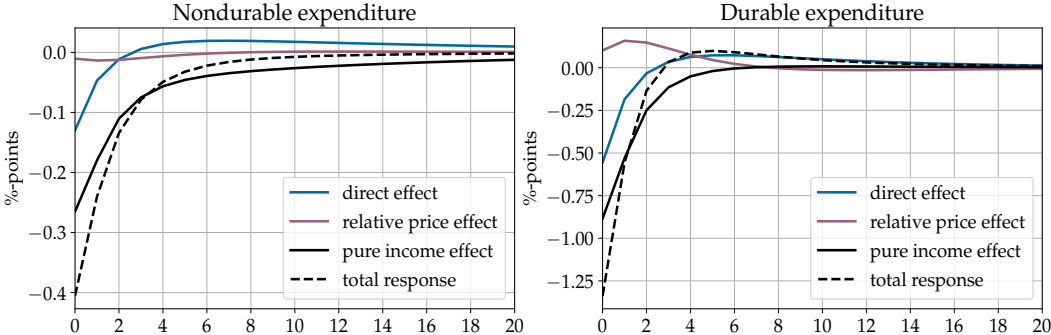
Figure 6: Expenditure response decomposition, robustness to different model alterations



(a) Baseline model



(b) Deficit financing



(c) Sticky wages

Note: Decomposition of the response of nondurable and durable expenditure into direct, relative-price and pure income effects. Top panel: baseline model; middle panel: model with deficit financing; bottom panel: model with sticky wages. We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

4.2 (which has been reproduced in the first row of the figure, to enhance comparability). Even with fiscal deficit financing, pure income effects drive the brunt of the response of both durables and nondurables. For a more disaggregated overview, we refer the reader to Figure 10 in Appendix F. As expected, taxes barely move in the presence of deficit financing.

4.4 Sticky wages

It is well understood that profit cyclicity may assume an important role in HANK settings. In our case, profits are countercyclical, in both sectors. This may lead some counterfactual effects in a setting with liquidity constrained agents (see, e.g., Broer et al., 2020). Thus, we want to test to which extent the qualitative and quantitative properties highlighted so far hold when limiting profit countercyclicity. This can be accomplished by introducing sticky wages. In addition, sticky wages reinforce comovement between durable and nondurable expenditure, as they dampen relative-price changes (see Carlstrom and Fuerst, 2010). The twist consists of replacing the wage schedule equation, (4), with a wage Phillips curve, as in Erceg et al. (2000), Erceg and Levin (2006) and Hagedorn et al. (2019). To this end, we assume that each household provides some (perfectly competitive) labor packers with differentiated labor services, so as to be transformed into aggregate effective labor. Thus, a union sells labor services at the nominal wage W_t (equalized across production sectors) to the labor recruiter, who minimizes costs given the aggregate demand for labor. In doing so, the union sets the nominal wage for one effective labor unit subject to virtual Rotemberg adjustment costs. Further analytical details about the modeling approach are reported in Appendix G.

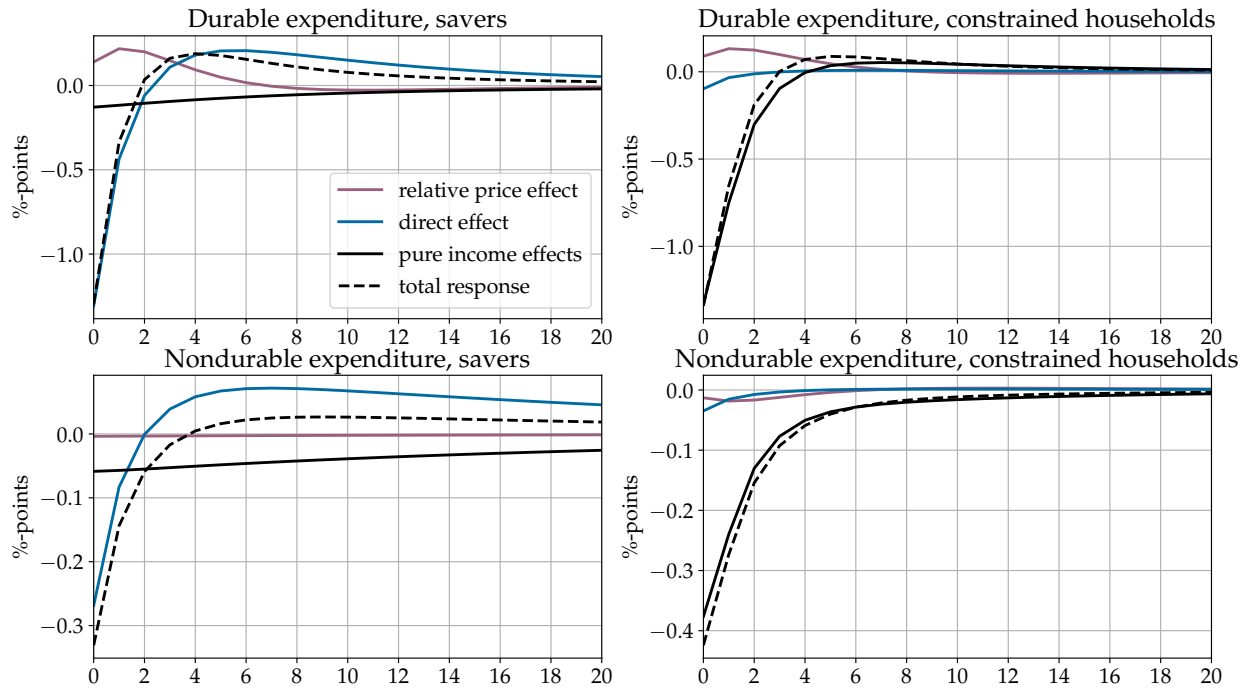
Re-calibration Given this extended structure, we need to re-calibrate some parameters. We set $\xi_n = 54.42$ and $\xi_d = 2.20$, such that the corresponding Calvo probabilities for prices are right on target (i.e, 0.75 and 0.25, respectively). As for wage stickiness, we set $\xi_w = 54.42$ to target a Calvo probability of 0.75, yielding an implied duration of wage contracts of one year, in line with the estimates of Smets and Wouters (2003) and Levin et al. (2005). We re-calibrate the parameter scaling the adjustment of durables, α , to 1.522, so as to target the relative (on-impact) volatility of $C_{d,t}$ to $C_{n,t}$. The model can now hit that target of 3.572 with precision. The borrowing wedge, κ , is re-calibrated to 0.0368, to target

a 30% steady-state share of liquidity-constrained households. The discount factor, β , is now 0.9634. The scaling of labor disutility, ψ_N , is 0.633. Finally, the implied steady-state total factor productivity in each sector are $A_n = 1$ and $A_d = 2.58$, while the steady-state nondurable-to-total consumption ratio equals 0.61 (so that $\gamma = 0.39$). Finally, we set the labor unions' market power in line with that characterizing the two intermediate-goods markets, so that $\epsilon_w = \epsilon_n = \epsilon_d = 0.6$.

Decomposition of expenditure responses The last row of Figure 6 reports the expenditure response decomposition for the model with sticky wages. In this case, pure income effects make up an even larger part of the response of both durables and nondurables. This is because prices inherit some stickiness from wages, causing relative-price movements to be smaller.¹¹ It should also be stressed that durables still display higher interest-rate sensitivity, though the gap of responsiveness with respect to nondurables along this dimension is heavily reduced, mostly because the durable figure is an order of magnitude lower, relative to its flexible-wage benchmark: over a year, the relative contribution of interest rate changes to the drop of private spending in either sector is 21% for nondurables and 37% for durables. The corresponding figures for pure income effects are 73% and 89%, respectively, while the relative-price effect alone accounts for 6% and -26% of either expenditure's total response, respectively. As for the response of total consumption, we have a contribution of 26%, 75% and -0.01% from direct, pure income and relative-price effects, respectively.

¹¹For a detailed account, see Appendix F, Figure 11.

Figure 7: Expenditure response decomposition by steady-state bond holdings (with sticky wages)



Note: Decomposition of nondurable and durable expenditure responses into direct, relative-price and pure income effects, for households differing with respect to their steady-state holdings of liquid assets (bonds). Liquidity-constrained households are defined as households with zero or less liquid assets. Savers are defined as households holding above zero liquid assets. The effects are calculated using the (initially) truncated distributions relative to a simulation of the relevant truncated distribution conditional on all input variables being at their steady-state values. We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

Importantly, the main takeaways from the liquidity-based decomposition in Section 4.2 carry over to the present setting, as implied by Figure 7: *i*) the aggregate relevance of pure income effects is mostly to be reconducted to the hand-to-mouth behavior of liquidity-constrained households, whose expenditures in either type of good comove positively; *ii*) interest-rate effects have strong grip on savers' expenditure in both types of good. In connection with this last observation, it is worth stressing that savers display positive comovement between durable and nondurable expenditure, in this model variation. This is because relative price movements have lost traction, due to sticky wages, and the direct channel of transmission of monetary disturbances becomes key in compressing savers' durable expenditure. More strikingly, savers and constrained households display remarkably similar total responses of both durable and nondurable spending, though it is worth stressing once again that the key channels of transmission are not the same, across agents sorted with respect to their holdings of liquid assets.

5 Concluding remarks

We have introduced durable goods into an otherwise standard New Keynesian model with heterogeneous households subject to idiosyncratic income risk, decomposing expenditure responses to a monetary policy shock into direct (interest-rate) and indirect (general-equilibrium) effects. Indirect effects are further decomposed into the response of spending in either type of good that can be ascribed to relative-price changes, and to general-equilibrium changes in other income components. Interest-rate effects make up a sizable fraction of the response of durables and, in turn, that of aggregate consumption. However, pure income effects dominate the responses of both types of expenditure. Moreover, pure income effects are key to undoing negative comovement that would otherwise stem from changes in the relative price of the two goods.

Despite the dominance of general-equilibrium effects, it is important to recall that, even when displaying similar reactivity with respect to their good-specific expenditures, savers and liquidity-constrained agents respond to fundamentally different incentives. In fact, while the former denote a strong attitude towards intertemporal substitution, the latter are extremely sensitive to pure income effects. This aspect, in conjunction with the dominant impact of durables on business-cycle volatility—and, more specifically, on the transmission of monetary shocks—renders the lessons learned in this paper rather relevant for deepening our understanding of monetary transmission, both in the dynamic and the cross-sectional dimension.

References

- Albouy, D., Ehrlich, G., and Liu, Y. (2016). Housing demand, cost-of-living inequality, and the affordability crisis. *NBER Working Paper Series*, pages 22816–.
- Alves, F., Kaplan, G., Moll, B., and Violante, G. L. (2020). A Further Look at the Propagation of Monetary Policy Shocks in HANK. *Journal of Money, Credit and Banking*, 52(S2):521–559.
- Attanasio, O. P. (1999). Consumption. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 11, pages 741–812. Elsevier.
- Auclert, A. (2019). Monetary Policy and the Redistribution Channel. *American Economic Review*, 109(6):2333–2367.
- Auclert, A., Bardóczy, B., and Rognlie, M. (2020a). MPCs, MPEs and Multipliers: A Trilemma for New Keynesian Models.
- Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models. *Econometrica*, 89(5):2375–2408.
- Auclert, A., Rognlie, M., and Straub, L. (2020b). Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model.
- Barsky, R. B., House, C. L., and Kimball, M. S. (2007). Sticky-Price Models and Durable Goods. *American Economic Review*, 97(3):984–998.
- Beraja, M. and Wolf, C. K. (2021). Demand Composition and the Strength of Recoveries.
- Berger, D. and Vavra, J. (2015). Consumption Dynamics During Recessions. *Econometrica*, 83(1):101–154.
- Bertola, G., Guiso, L., and Pistaferri, L. (2005). Uncertainty and Consumer Durables Adjustment. *The Review of Economic Studies*, 72(4):973–1007.
- Broer, T., Hansen, N.-J. H., Krusell, P., and Öberg, E. (2020). The new keynesian transmission mechanism: A heterogeneous-agent perspective. *IDEAS Working Paper Series from RePEc*, 87(1):77–101.

- Carlstrom, C. and Fuerst, T. (2010). Nominal rigidities, residential investment, and adjustment costs. *Macroeconomic Dynamics*, 14(1):136–148.
- Davis, M. A. and Ortalo-Magné, F. (2011). Household expenditures, wages, rents. *Review of economic dynamics*, 14(2):248–261.
- Debortoli, D. and Galí, J. (2021). Monetary policy with heterogeneous agents: Insights from TANK models. Technical Report 1686, Department of Economics and Business, Universitat Pompeu Fabra. Publication Title: Economics Working Papers.
- Dey, J. and Tsai, Y.-C. (2012). Explaining the durable goods co-movement puzzle with non-separable preferences: a bayesian approach. Technical Report 57805, University Library of Munich, Germany. Publication Title: MPRA Paper.
- Erceg, C. and Levin, A. (2006). Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53(7):1341–1359.
- Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281–313.
- Hagedorn, M., Manovskii, I., and Mitman, K. (2019). The Fiscal Multiplier.
- Harmenberg, K. and Öberg, E. (2021). Consumption dynamics under time-varying unemployment risk. *Journal of Monetary Economics*, 118:350–365.
- Iacoviello, M. (2005). House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95(3):739–764.
- Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary Policy According to HANK. *American Economic Review*, 108(3):697–743.
- Kaplan, G., Violante, G. L., and Weidner, J. (2014). The Wealthy Hand-to-Mouth. *Brookings Papers on Economic Activity*, 45(1 (Spring)):77–153.
- Katayama, M. and Kim, K. H. (2013). The delayed effects of monetary shocks in a two-sector New Keynesian model. *Journal of Macroeconomics*, 38:243–259.
- Klenow, P. J. and Malin, B. A. (2010). Microeconomic Evidence on Price-Setting. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 6, pages 231–284. Elsevier.

- Laibson, D., Maxted, P., and Moll, B. (2022). A simple mapping from mpcs to mpxs. *NBER Working Paper Series*.
- Levin, A. T., Onatski, A., Williams, J. C., and Williams, N. (2005). Monetary Policy under Uncertainty in Micro-Founded Macroeconometric Models. *NBER Macroeconomics Annual*, 20:229–287. Publisher: University of Chicago Press.
- Mankiw, N. G. (1985). Consumer Durables and the Real Interest Rate. *The Review of Economics and Statistics*, 67(3):353–362.
- McKay, A., Nakamura, E., and Steinsson, J. (2016). The Power of Forward Guidance Revisited. *American Economic Review*, 106(10):3133–3158.
- McKay, A. and Wieland, J. F. (2021). Lumpy Durable Consumption Demand and the Limited Ammunition of Monetary Policy. *Econometrica*, 89(6):2717–2749.
- McKay, A. and Wieland, J. F. (2022). Forward Guidance and Durable Goods Demand. *American Economic Review: Insights*, 4(1):106–122.
- Monacelli, T. (2009). New Keynesian models, durable goods, and collateral constraints. *Journal of Monetary Economics*, 56(2):242–254.
- Nakamura, E. and Steinsson, J. (2008). Five Facts about Prices: A Reevaluation of Menu Cost Models*. *The Quarterly Journal of Economics*, 123(4):1415–1464.
- Ogaki, M. and Reinhart, C. (1998). Measuring intertemporal substitution: The role of durable goods. *The Journal of political economy*, 106(5):1078–1098.
- Pakos, M. (2011). Estimating intertemporal and intratemporal substitutions when both income and substitution effects are present: The role of durable goods. *Journal of business economic statistics*, 29(3):439–454.
- Petrella, I., Rossi, R., and Santoro, E. (2019). Monetary Policy with Sectoral Trade-Offs. *Scandinavian Journal of Economics*, 121(1):55–88.
- Smets, F. and Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175.
- Sterk, V. and Tenreyro, S. (2018). The transmission of monetary policy through redistributions and durable purchases. *Journal of Monetary Economics*, 99:124–137.

- Stock, J. H. and Watson, M. W. (1999). Business cycle fluctuations in us macroeconomic time series. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 1, pages 3–64. Elsevier.
- Sudo, N. (2012). Sectoral Comovement, Monetary Policy Shocks, and Input-Output Structure. *Journal of Money, Credit and Banking*, 44(6):1225–1244.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.
- Tsai, Y.-C. (2016). What Do Working Capital And Habit Tell Us About The Co-Movement Problem? *Macroeconomic Dynamics*, 20(1):342–361.
- Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control*, 34(1):36–41.

Appendix

A Endogenous grid method

A.1 Model setup

Households face the following optimization problem:

$$\begin{aligned}
 V_t(z_t, b_t, d_t) &= \max_{c_t, d_{t+1}, b_{t+1}} u(c_t, d_t) + \beta \mathbb{E}_t V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}) \\
 \text{s.t. } c_t + b_{t+1} + Q_t(d_{t+1} - (1 - \delta)d_t) &= z_t + (1 + r_t)b_t - \Psi(d_{t+1}, d_t) \\
 b_t &\geq \underline{b}, \quad d_t \geq 0,
 \end{aligned} \tag{22}$$

where z_t denotes idiosyncratic income, b_t is wealth, d_t denotes the stock of durables and Q_t is the price of durables relative to that of nondurables. In the general equilibrium setting, $z_t = \exp\{e_t\} [w_{n,t}N_t - \tau_t + Div_t]$. The rest, except for utility and the cost function $\Psi(\cdot)$ is standard. The utility and the adjustment cost functions are

$$\begin{aligned}
 u(c_t, d_t) &= \frac{\psi(c_t, d_t)^{1-\sigma}}{1-\sigma} \quad \text{and} \quad \psi(c_t, d_t) = c_t^\theta d_t^{1-\theta}, \\
 \Psi(d_{t+1}, d_t) &= \frac{\alpha}{2} \left(\frac{d_{t+1} - (1-\delta)d_t}{d_t} \right)^2 d_t.
 \end{aligned} \tag{23}$$

A.2 First-order and envelope conditions

Re-write the Bellman equation by substituting out consumption using the budget constraint

$$\begin{aligned}
 V_t(z_t, b_t, d_t) &= \max_{b_{t+1}, d_{t+1}} u(z_t + (1 + r_t)b_t - Q_t(d_{t+1} - (1 - \delta)d_t) - \Psi(d_{t+1}, d_t) - b_{t+1}, d_t) \\
 &\quad + \mu_t d_{t+1} + \lambda_t (b_{t+1} - \underline{b}) + \beta \mathbb{E}_t V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}),
 \end{aligned} \tag{24}$$

where μ_t and λ_t are the multipliers for the non-negativity constraint on durables and the unsecured credit-borrowing constraint, respectively.

The first-order conditions with respect to d_{t+1} and b_{t+1} yield

$$\begin{aligned}\partial_{c_t} u(c_t, d_t) (Q_t + \partial_{d_{t+1}} \Psi(d_{t+1}, d_t)) &= \mu_t + \partial_{d_{t+1}} \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}), \\ \partial_{c_t} u(c_t, d_t) &= \lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}).\end{aligned}\quad (25)$$

The envelope conditions are

$$\begin{aligned}\partial_{b_t} V_t(z_t, b_t, d_t) &= (1 + r_t) \partial_c u(c_t, d_t), \\ \partial_{d_t} V_t(z_t, b_t, d_t) &= \partial_{d_t} u(c_t, d_t) + \partial_{c_t} u(c_t, d_t) [Q(1 - \delta) - \partial_{d_t} \Psi(d_{t+1}, d_t)].\end{aligned}\quad (26)$$

For later use, it is convenient to define the post-decision value function as

$$W_t(z_t, b_{t+1}, d_{t+1}) \equiv \beta \mathbb{E}_t V_{t+1}(z_t, b_{t+1}, d_{t+1}). \quad (27)$$

A.3 Main equations of the algorithm

First, we combine the two equations in (25) to obtain

$$\frac{\mu_t + \partial_d \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1})}{\lambda_t + \partial_b \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1})} = Q_t + \alpha \left(\frac{d_{t+1}}{d_t} - (1 - \delta) \right). \quad (28)$$

From the F.O.C. wrt. b_{t+1} in eq. (25) we can pin down nondurable consumption:

$$\begin{aligned}\frac{\partial u(c_t, d_t)}{\partial c_t} &= \lambda_t + \partial_{a_{t+1}} \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}) \\ \Rightarrow \theta c_t^{\theta-1} d_t^{1-\theta} [c_t^\theta d_t^{1-\theta}]^{-\sigma} &= \lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}) \\ \Rightarrow c_t &= \left[\frac{1}{\theta} (\lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1})) d_t^{(\theta-1)(1-\sigma)} \right]^{\frac{1}{\theta(1-\sigma)-1}}.\end{aligned}\quad (29)$$

A.4 Algorithm

The algorithm is based on the two-asset algorithm described in Auclert et al. (2021). For a generic variable x_t , denote today's grid by x and tomorrow's grid by x' . Thus, according to the EGM algorithm:

1. When seeking for steady-state policies, initialize the guess on $\partial_b V(z, b, d)$, $\partial_d V(z, b, d)$. Otherwise, start backward induction by starting from steady-state $\partial_b V(z, b, d)$, $\partial_d V(z, b, d)$ (used when calculating household Jacobians).
2. Let the productivity-shock transmission matrix be notated by Π . The value functions have a common $z' \rightarrow z$ so the post-decision functions are:

$$\begin{aligned} W_b(z, b', d') &= \beta \Pi V_b(z', b', d'), \\ W_d(z, b', d') &= \beta \Pi V_d(z', b', d'). \end{aligned} \quad (30)$$

3. Find $d'(z, b', d)$ for the *unconstrained* case using eq. (28):

$$\frac{W_d(z, b', d')}{W_b(z, b', d')} = Q + \alpha \left(\frac{d'}{d} - (1 - \delta) \right). \quad (31)$$

4. Use $d'(z, b', d)$ to map $W_b(z, b', d')$ into $W_b(z, b', d)$ by interpolation. Then compute consumption by using eq. (29):

$$c(z, b', d) = \left(W_b(z, b', d) d^{\theta-1} \cdot d^{(1-\theta)\sigma} \right)^{\frac{1}{\theta(1-\sigma)-1}}. \quad (32)$$

5. Now it is possible to find total assets by inserting $d'(z, b', d)$ and $c(z, b', d)$ into the budget constraint:

$$b(z, b', d) = \frac{c(z, b', d) + Q(d'(z, b', d) - (1 - \delta)d) + b' + \Psi(d'(z, b', d), d) - z}{1 + r}. \quad (33)$$

6. Invert $b(z, b', d)$ to obtain $b'(z, b, d)$ by interpolation. Use the same interpolation weights to obtain $d'(z, b, d)$.
7. Find $d'(z, \underline{b}, d)$ for the *constrained* case using eq. (28). For scaling, define $\kappa \equiv \lambda/W_b(z, \underline{b}, d')$. Then eq. (28) becomes

$$\frac{1}{1 + \kappa} \frac{W_d(z, \underline{b}, d')}{W_b(z, \underline{b}, d')} = Q + \alpha \left(\frac{d'}{d} - (1 - \delta) \right). \quad (34)$$

8. Use eq. (34) to solve for $d'(z, \kappa, d)$, that is over a grid of κ values. Then compute

consumption as

$$c(z, \kappa, d) = ((1 + \kappa)W_b(z, \kappa, d) d^{\theta-1} \cdot d^{(1-\theta)\sigma})^{\frac{1}{\theta(1-\sigma)-1}}. \quad (35)$$

9. Using $d'(e, \kappa, d)$, $c(e, \kappa, d)$ and the budget constraint obtain

$$b(z, \kappa, d) = \frac{c(z, \kappa, d) + Q(d'(z, \kappa, d) - (1 - \delta)d) + \underline{b} + \Psi(d'(z, \kappa, d), d) - z}{1 + r}. \quad (36)$$

10. Invert $b(z, \kappa, d)$ by interpolation to obtain $\kappa(z, b, d)$. The same interpolation weights can be used to map $d'(z, \kappa, d)$ into $d'(z, b, d)$. By definition, $b'(z, b, d) = \underline{b}$.

11. Combine the constrained and the unconstrained solutions of $b'(z, b, d)$ and $d'(z, b, d)$. Then compute consumption from the budget constraint:

$$c(z, b, d) = z + (1 + r)b - Q(d'(z, b, d) - (1 - \delta)d) - \Psi(d', d) - b'(z, b, d). \quad (37)$$

12. Update $\partial_b V(z, b, d)$ and $\partial_d V(z, b, d)$ using the envelope conditions from eq. (26):

$$\begin{aligned} \partial_b V(z, b, d) &= (1 + r) \partial_c u(c, d), \\ \partial_d V(z, b, d) &= \partial_d u(c, d) - \partial_c u(c, d) [Q(1 - \delta) + \partial_d \Psi(d', d)]. \end{aligned} \quad (38)$$

13. For the steady-state solutions: Return to step 2 and follow the same steps until the change in $\partial_b V(z, b, d)$ and $\partial_d V(z, b, d)$ between iterations is ≈ 0 . Otherwise, solve paths by backward iteration (used to obtain household Jacobians given some shock to a given household input variable).

Finally, to obtain aggregates we need to simulate the distribution of households. We use the histogram method as developed in Young (2010). In the steady state, we simulate forward until the change in the distribution between consecutive iterations is ≈ 0 (see Appendix B). Outside the steady state, one can simply simulate forward given a path length (used to obtain Jacobians).

B Deterministic steady state

The distribution is obtained by relying on the deterministic histogram method of Young (2010). Given guesses for β, Q, N_d , we can solve for equilibrium quantities as follows:

1. We set $P_n = 1$ as the numeraire, so that $\Pi_n = 1$;
2. We get that $\Pi_d = 1$, as $\Pi_d = \Pi_n$ in the steady state;
3. Given a calibration target for Y_d (which is set to 0.5), we pin down $A_d = Y_d/N_d$ ¹²;
4. We obtain $w_d = A_d \cdot \frac{\epsilon_d - 1}{\epsilon_d}$ from the durable-goods sector Phillips curve;
5. The latter then yields real wage in the nondurable-goods sector as $w_n = Q \cdot w_d$, as the nominal wage is equalized across sectors;
6. From the nondurable-goods sector Phillips curve we can pin down $A_n = w_n \cdot \frac{\epsilon_n}{\epsilon_n - 1}$;
7. We set $Y_n = 1 - Q \cdot Y_d$, such that total output, $Y = 1$;
8. We then obtain employment in the nondurable-goods sector as $N_n = Y_n/A_n$;
9. We get dividends from eq. (11), $Div(Y_n, Y_d, Q, w_n, w_d)$;
10. Taxes are pinned down as $\tau = r \cdot B^g$.

As we pin down all variables from aggregate relationships, it is then possible to solve the household problem to obtain C_n, C_d, B , and check root-finding target residuals. Thus, after root-finding, we set ψ_N given w_n, C_n, C_d and the parameters, such that the wage schedule, eq. (4), holds in the steady state.

¹² $Y_d = 0.5$ is a reasonable choice—given that $Y_d = C_d$ —as C_d makes up a empirically plausible share of total consumption; cf. the calibration target for $C_n/(C_n + C_d)$.

C Sequence space formulation for impulse responses

In sequence space, the model can be summarized by the equation system

$$H(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r) = \begin{pmatrix} \text{Wage schedule} \\ \text{NKPC durables} \\ \text{NKPC nondurables} \\ \text{Bonds market} \\ \text{Goods market durables} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (39)$$

Denoting the aggregate solution variables with $\mathcal{B}, \mathcal{C}_n, \mathcal{C}_d, \mathcal{D}$, the system can be reported as

$$H(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r) = \begin{pmatrix} w_{n,t} - \psi_N N_t^{\frac{1}{\theta}} (\mathcal{C}_{n,t}^\theta \mathcal{D}_t^{1-\theta})^\sigma \left(\frac{\mathcal{C}_{n,t}}{\mathcal{D}_t}\right)^{1-\theta} \\ (1 - \epsilon_d) + \epsilon_d w_{d,t}/A_n - \xi_d (\Pi_{d,t} - 1) \Pi_{d,t} + \beta \xi_d (\Pi_{d,t+1} - 1) \Pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \\ (1 - \epsilon_n) + \epsilon_n w_{n,t}/A_d - \xi_n (\Pi_{n,t} - 1) \Pi_{n,t} + \beta \xi_n (\Pi_{n,t+1} - 1) \Pi_{n,t+1} \frac{Y_{n,t+1}}{Y_{n,t}} \\ \mathcal{B}_t - B^g \\ Y_{d,t} - \mathcal{C}_{d,t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (40)$$

where we have

$$\Pi_{d,t} = \frac{Q_t}{Q_{t-1}} \Pi_{n,t} \quad (41)$$

$$Y_{n,t} = A_n N_{n,t} \quad (42)$$

$$Y_{d,t} = A_d N_{d,t} \quad (43)$$

$$N_t = N_{n,t} + N_{d,t} \quad (44)$$

$$w_{d,t} = Q_t^{-1} w_{n,t} \quad (45)$$

$$Div_t = Y_{n,t} - w_{n,t} N_{n,t} + Q_t [Y_{d,t} - w_{d,t} N_{d,t}] \quad (46)$$

$$\tilde{\Pi}_t = \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^\gamma \quad (47)$$

$$i_t = u_t^r + \phi_{\tilde{\pi}} \tilde{\pi}_t \quad (48)$$

$$r_t = \frac{1 + i_{t-1}}{1 + \pi_{n,t}} - 1 \quad (49)$$

$$\tau_t = r_t B_g \quad (50)$$

and where the market for nondurable goods clears by Walras' law.

D Sequence space formulation with deficit financing

All targets and variables stay the same as in Appendix C. The only difference is that we replace equation (50) with

$$\tau_t = \tau + \phi_\tau (B_{t-1}^g - B^g), \quad (51)$$

where it has to hold that

$$(1 + r_t) B_{t-1}^g = \tau_t + B_t^g. \quad (52)$$

Thus, we use a root-finder to solve for the path of B_t^g consistent with eq. (52), nested in the sequence space formulation. For further details, see Appendix C.5 in Auclert et al. (2021). The model can then be solved in sequence space, as described in Appendix C.

E Sequence space formulation with sticky wages

In sequence space, the model with the wage Phillips curve can be summarized by the equation system

$$H(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r) = \begin{pmatrix} \text{Wage Phillips curve} \\ \text{Phillips curve durables} \\ \text{Phillips curve nondurables} \\ \text{Bonds market clearing} \\ \text{Durable goods market clearing} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (53)$$

Using caligraphic variables $\mathcal{B}, \mathcal{C}_n, \mathcal{C}_d, \mathcal{D}$ to denote the aggregated household solution variables counterparts, the system reads as

$$\begin{pmatrix} H(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r) = \\ \left(\begin{array}{l} (1 - \epsilon_w) w_{n,t} + \epsilon_w \frac{U'_N(N_t)}{U'_{C_n}(C_{n,t}, D_t)} - \xi_w (\Pi_{w,t} - 1) \Pi_{w,t} + \beta \xi_w (\Pi_{w,t+1} - 1) \Pi_{w,t+1} \frac{N_{t+1}}{N_t} \\ (1 - \epsilon_d) + \epsilon_d w_{d,t} / A_n - \xi_d (\Pi_{d,t} - 1) \Pi_{d,t} + \beta \xi_d (\Pi_{d,t+1} - 1) \Pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \\ (1 - \epsilon_n) + \epsilon_n w_{n,t} / A_d - \xi_n (\Pi_{n,t} - 1) \Pi_{n,t} + \beta \xi_n (\Pi_{n,t+1} - 1) \Pi_{n,t+1} \frac{Y_{n,t+1}}{Y_{n,t}} \\ \mathcal{B}_t - B^g \\ Y_{d,t} - \mathcal{C}_{d,t} \end{array} \right) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{pmatrix} \quad (54)$$

where we have

$$\Pi_{d,t} = \frac{Q_t}{Q_{t-1}} \Pi_{n,t} \quad (55)$$

$$\Pi_{w,t} = \frac{w_{n,t}}{w_{n,t-1}} \cdot \Pi_{n,t} \quad (56)$$

$$Y_{n,t} = A_n N_{n,t} \quad (57)$$

$$Y_{d,t} = A_d N_{d,t} \quad (58)$$

$$N_t = N_{n,t} + N_{d,t} \quad (59)$$

$$w_{d,t} = Q_t^{-1} w_{n,t} \quad (60)$$

$$Div_t = Y_{n,t} - w_{n,t} N_{n,t} + Q_t [Y_{d,t} - w_{d,t} N_{d,t}] \quad (61)$$

$$\tilde{\Pi}_t = \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^\gamma \quad (62)$$

$$i_t = u_t^r + \phi_{\tilde{\pi}} \tilde{\pi}_t \quad (63)$$

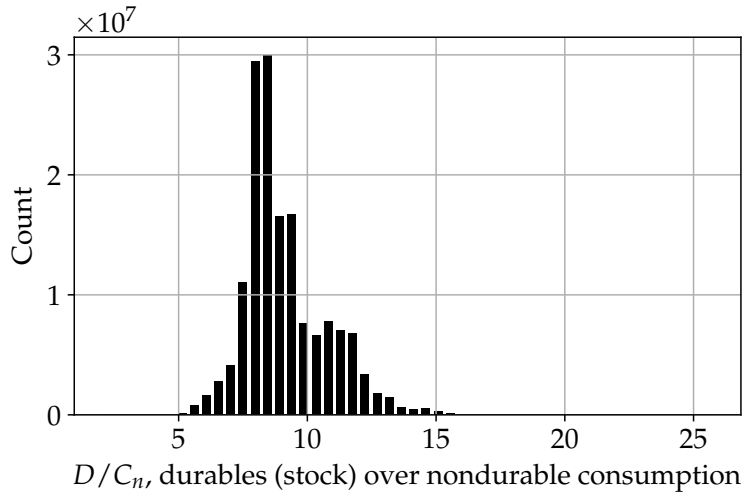
$$r_t = \frac{1 + i_{t-1}}{1 + \pi_{n,t}} - 1 \quad (64)$$

$$\tau_t = r_t B_g \quad (65)$$

and where the nondurable goods market clears by Walras' law.

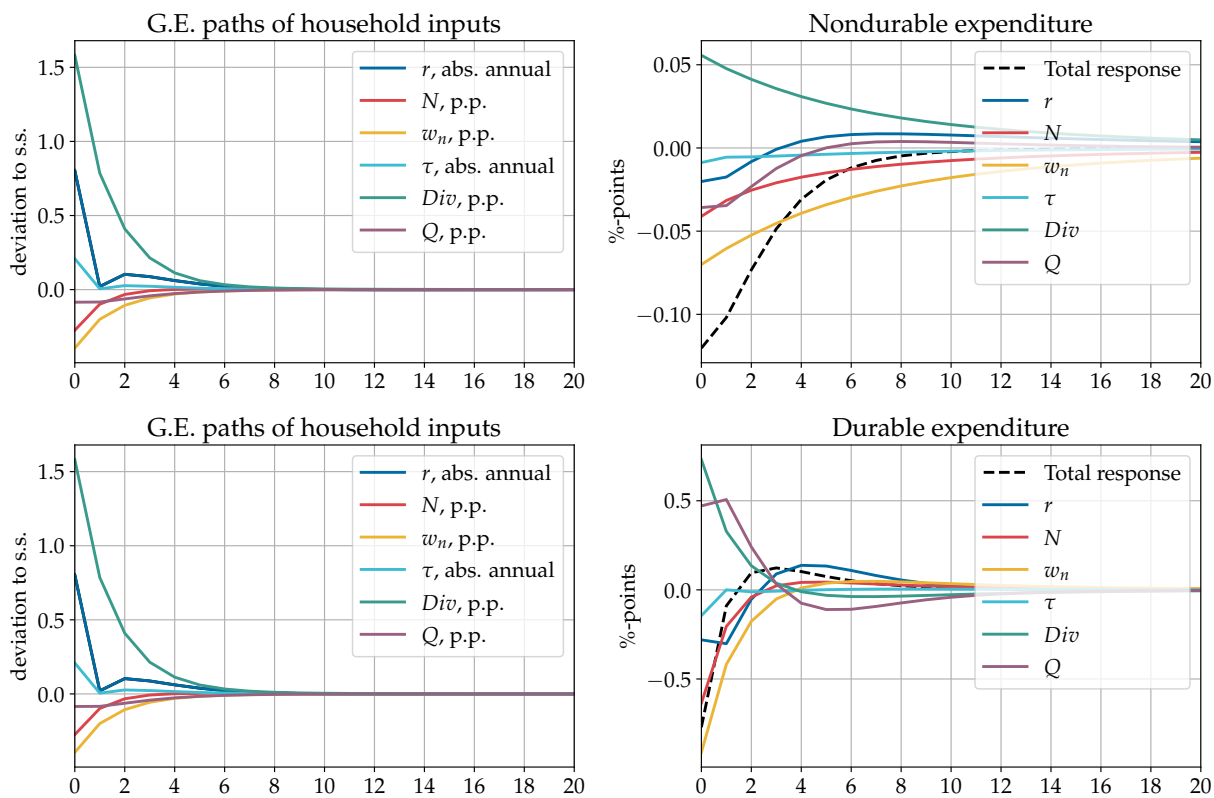
F Additional figures

Figure 8: Histogram of the ratio between the steady-state stock of durables and non-durable consumption



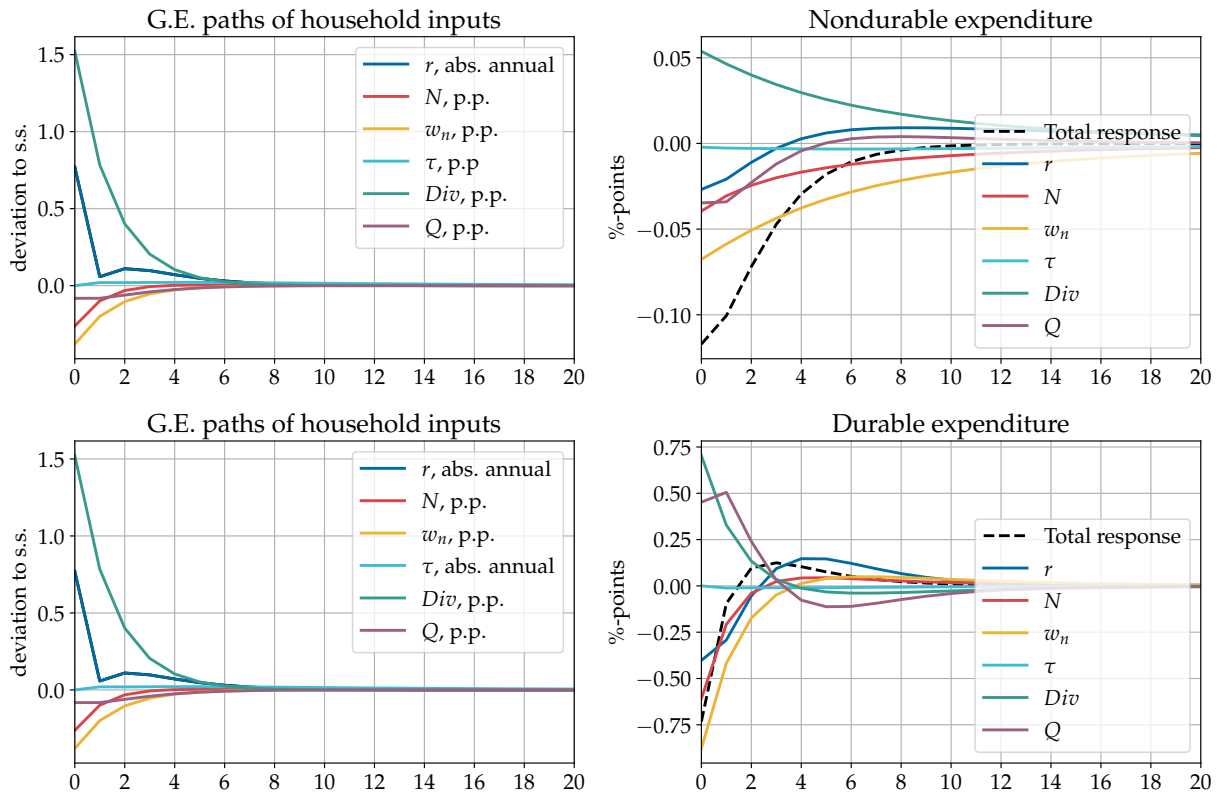
Note: To generate the histogram, we Monte Carlo simulate the steady-state household distribution using 2D linear interpolation over the policy functions. We simulate 80.000 households for 2.000 periods, and discard the first 1.000 periods. We use 50 bins for plotting.

Figure 9: Detailed expenditure response decomposition



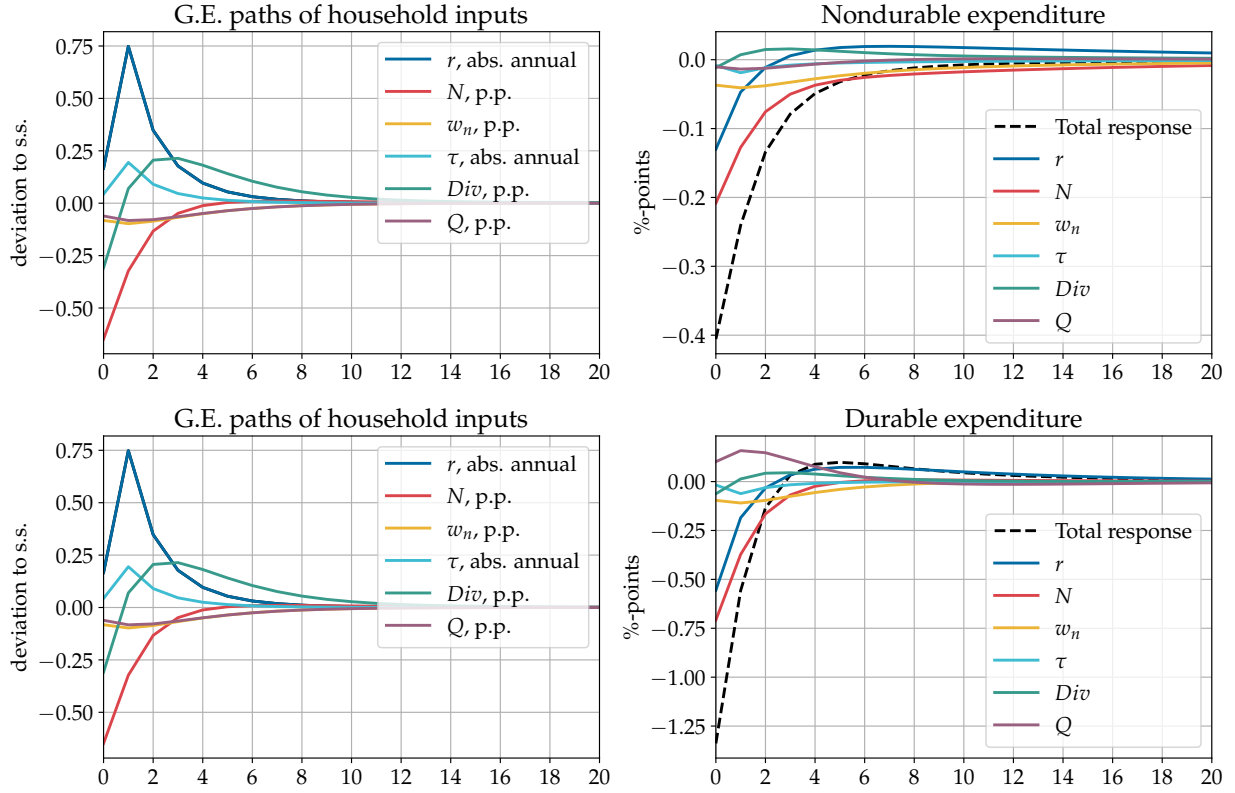
Note: Absolute annual deviations are calculated for visualization purposes.

Figure 10: Detailed expenditure response decomposition under deficit financing



Note: Absolute annual deviations are calculated for visualization purposes.

Figure 11: Detailed expenditure response decomposition under sticky wages



Note: Absolute annual deviations are calculated for visualization purposes.

G Model with sticky wages

We replace the wage schedule equation, eq. (4), with a wage Phillips curve, in the vein of Erceg et al. (2000), Erceg and Levin (2006) and Hagedorn et al. (2019). Specifically, each household provides differentiated labor services, which are transformed into aggregate effective labor, N_t , by perfectly competitive labor packers, using the technology

$$N_t = \left(\int_0^1 \exp\{e(s)_t\} (\mathcal{N}(s)_t)^{\frac{\epsilon_w - 1}{\epsilon_w}} ds \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}. \quad (66)$$

A union sells labor services at the nominal wage W_t (equalized across production sectors) to the labor recruiter, who minimizes costs given the aggregate demand for labor, implying

$$\mathcal{N}(s)_t = \mathcal{N}(W(s)_t; W_t, N_t) = \left(\frac{W(s)_t}{W_t} \right)^{-\epsilon_w} N_t \quad (67)$$

for the s th household, and where the equilibrium nominal wage amounts to

$$W_t = \left(\int_0^1 \exp\{e(s)_t\} W(s)_t^{1-\epsilon_w} ds \right)^{\frac{1}{1-\epsilon_w}}. \quad (68)$$

The union sets the nominal wage for one effective labor unit, \hat{W}_t , such that $\hat{W}_t = W_t$ subject to virtual Rotemberg adjustment costs:

$$\mathcal{C}_w(\cdot) = \exp\{e(s)_t\} \frac{\xi_w}{2} \left(\frac{W_{it}}{W_{it-1}} - 1 \right)^2 N_t, \quad (69)$$

assuming steady-state $\Pi_w = 1$. The union's wage-setting problem maximizes

$$\begin{aligned} V_t^w(\hat{W}_{t-1}) \equiv \max_{\hat{W}_t} \int \frac{\exp\{e(s)_t\} (1 - \tau_t) \hat{W}_t}{P_{n,t}} \mathcal{N}(\hat{W}_t; W_t, N_t) - \frac{v(\mathcal{N}(\hat{W}_t; W_t, N_t))}{U'_{C_n}(C_{n,t}, D_t)} ds \\ - \int \exp\{e(s)_t\} \frac{\xi_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - 1 \right)^2 N_t ds + \beta V_{t+1}^w(\hat{W}_t). \end{aligned}$$

This problem yields a wage Phillips curve:¹³

$$(1 - \epsilon_w) w_{n,t} + \epsilon_w \frac{U'_N(N_t)}{U'_{C_n}(C_{n,t}, D_t)} - \xi_w (\Pi_{w,t} - 1) \Pi_{w,t} + \beta \xi_w (\Pi_{w,t+1} - 1) \Pi_{w,t+1} \frac{N_{t+1}}{N_t} = 0, \quad (70)$$

where the aggregation assumptions are as in Hagedorn et al. (2019), so that one obtains the RA outcome as heterogeneity is turned off.

The steady state is solved as described in Appendix B: however, instead of varying ψ_N such that the wage schedule, eq. (4), holds in the steady state, we vary it to ensure that the steady-state wage Phillips curve holds. As for the dynamic solution, we refer the reader to Appendix E.

¹³See Hagedorn et al. (2019).