Monetary Policy in Investment Networks*

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Abstract

The investment network, which links industries through firms' sourcing of capital goods, shapes monetary transmission. Using U.S. data, we document that investment-upstream sectors experience larger output sensitivity to monetary policy shocks. In a multi-sector New Keynesian economy, monetary policy shocks alter investment orders; the resulting changes in demand cascade through capital-goods suppliers, magnifying output responses upstream. The network also matters for optimal policy. While input–output linkages call for leaning against upstream inflation, investment linkages call for accommodating it: shocks to investment hubs raise the user cost of capital.

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1 Introduction

Capital formation is a key channel of monetary transmission (Christiano et al., 2005; Ottonello and Winberry, 2020). Typically, it is modeled as the accumulation of a single aggregate good. However, most investment goods are produced by a few investment hubs and installed as capital in other sectors (vom Lehn and Winberry, 2022). These bilateral capital-goods flows form the *investment network*—the investment analogue of a production network: whereas production networks map intermediate inputs into current production, the investment network maps capital goods into future productive capacity. Yet, little is known about how this structure shapes monetary transmission. This paper shows empirically, analytically, and quantitatively that the investment network governs how monetary policy propagates across sectors and alters the optimal policy response to sectoral disturbances.

We start by documenting that the investment network shapes the transmission of monetary policy shocks across sectors. We use quarterly data on 66 private U.S. industries, corresponding to three-digit NAICS codes, and consider the monetary policy shocks of Bauer and Swanson (2023). The BEA Capital Flow Table allows us to measure each industry's *investment upstreamness*, which captures the extent to which it supplies capital goods to other sectors, both directly and indirectly.

We then estimate panel local projections of sectoral real gross output and inflation on the interaction between monetary policy shocks and investment upstreamness. To establish the relevance of the investment network beyond the production network, we also include the interaction between the shock and input–output upstreamness. The specification features industry-by-quarter-of-year fixed effects and 2-digit NAICS-sector-by-quarter fixed effects, which capture industry-specific seasonal patterns and any time-varying factor that is common to industries of the same sector. This setting absorbs the aggregate effect of the monetary policy shock, but allows us to cleanly identify how its transmission varies across sectors depending on their position in the investment network, that is, the *differential* sectoral transmission of monetary policy.

Following a 25 basis-point (bp) interest-rate hike, real gross output contracts more in investment-upstream industries: at the peak response, a one-standard-deviation increase in investment upstreamness deepens this contraction by about 1.02 percentage points. This differential is statistically significant, and almost matches the effect associated with input–output upstreamness. The investment network thus does shape the real effects of monetary policy across industries. On the other hand, inflation differentials are very limited and not statistically significant.

To rationalize this evidence, we extend an otherwise standard multi-sector New Keynesian model with an investment network. Sectors differ in their factor intensities, their contribution to consumption, their degree of price rigidity, and their position in

the input–output network. In addition, each sector accumulates its own capital stock by combining capital goods produced across industries. The exact sectoral composition of these purchases defines the investment network.

To uncover the monetary transmission mechanism, we characterize analytically the response of value added in a simplified version of the model, in which sectors are identical in every dimension except their position in the investment network. After a monetary tightening, the cumulative contraction of real value added is strictly larger the more investment-upstream a sector is. Two channels deliver this result.

The first is a *final-demand* channel. A monetary tightening lowers consumption, and the real wage falls with it. Cheaper labor leads producers to hold less capital at any level of output, so they place fewer orders with the industries that supply their capital goods. The resulting decline in sales is larger the more investment-upstream a sector is. The second is a *user-cost* channel. The tightening raises the real interest rate and, with it, the user cost of capital, so every sector shrinks its desired capital stock and cuts investment. This cut in capital spending falls on the producers of capital goods, and again hits hardest those that are most investment-upstream.

We calibrate the model to the U.S. economy, with the same 66 industries as in the empirical analysis. We discipline most of the sectoral heterogeneity with information from the Input–Output Tables of the U.S. Bureau of Economic Analysis (BEA), which pins down sectors' factor intensities, contribution to consumption, and position in the input–output network. The heterogeneity in price rigidity reflects the sectoral price durations documented by Pasten et al. (2020). Then, sectoral depreciation rates come from the BEA Fixed Assets Tables. Finally, we build the investment network following the capital-flow methodology of vom Lehn and Winberry (2022), which maps each sector's investment expenditures into the sectors that produce the corresponding capital goods.

To assess whether our economy can account for the patterns we document in the data, we examine how the sectoral gross-output and inflation responses to a 25 bp interest-rate hike vary with investment upstreamness in the model. We find that a one-standard-deviation increase in investment upstreamness deepens the peak contraction in gross output by 0.92 percentage points, nearly matching the empirical estimate of 1.02 percentage points. When we abstract from heterogeneity in the investment network and make it homogeneous across industries, this relationship disappears. Consistent with the data, the inflation response instead varies little with investment upstreamness. The model's ability to match these empirical findings makes it an ideal laboratory for studying the implications of the investment network for monetary policy.

We then use the model to quantify the contribution of the investment network to the aggregate effects of monetary policy. While it is well established that sectoral heterogeneity amplifies the real effects of monetary policy (Pasten et al., 2020), we ask how much of this amplification comes from the investment network. We first compare the fully heterogeneous

calibration with a fully homogeneous economy to measure the amplification due to sectoral heterogeneity. We then shut down heterogeneity in the investment network, keeping all other sources of heterogeneity unchanged, and measure how much of the amplification disappears. The investment network accounts for 70 percent of the additional GDP response after one quarter, and 63 percent after one year. Instead, there is no effect on the inflation amplification. Thus, the investment network is a first-order determinant of the real amplification associated with sectoral heterogeneity.

Having studied how the investment network shapes the transmission of monetary policy, we turn to its normative implications. We do so by characterizing optimal policy under commitment upon sectoral cost-push shocks. The optimal interest rate features two components. The first reflects the commitment of the planner to stabilize prices, taking into account both the resource cost of inflation and the effect of input prices on the factor mix. As in production networks, the planner assigns a larger weight to the inflation of sectors with a larger social value (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). The investment network shapes these weights because the sales of a sector include the capital goods of its customers. This component remains when capital fully depreciates and, with contemporaneous installation, coincides with the policy rule in an economy with labor and intermediate inputs.

The second component captures the additional role of durability. Since part of the capital stock survives into the next period, its user cost depends on the interest rate and on expected capital gains. A shock to an investment hub raises the price of capital goods for its customers. For a sufficiently small capital share, the user-cost component calls for accommodating more the shocks that hit sectors with a larger social value. The price-stabilization component can instead call for leaning more against these shocks; we state the condition under which the two components move in opposite directions. Thus, durability can overturn the tighter response associated with price stabilization.

We then proceed to quantify the optimal policy stance upon sectoral inflationary shocks. To put the relevance of the investment network into context, we compare its prescriptions with those of the production network. In this case, we confirm the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023): the planner tightens more upon shocks that hit upstream sectors. We set up the exercise by normalizing shock sizes so that a shock to the most upstream sector raises the real interest-rate gap, that is, the deviation of the real interest rate from its flex-price value, by 100 bp. We find that shocks to the most downstream sector push the planner to accommodate, lowering the rate gap by 23 bp. Hence, going from the most downstream to the most upstream sector raises the response of the rate gap by 123 bp. The investment network delivers the opposite prescription: the planner accommodates upstream shocks. Going from the most downstream to the most upstream sector, the rate gap falls from 16 to -4 bp. The differential is thus more limited than with the production network, at 20 bp.

Hence, the investment network overturns the prescription that input–output linkages deliver for the conduct of monetary policy across sectors: the central bank should accommodate, rather than lean against, the shocks that hit upstream sectors, that is, those that produce the capital goods of the economy. These different prescriptions also matter for welfare: ignoring investment linkages generates losses ranging from 75 to 197 percent of those from ignoring input–output linkages (depending on the source of the shocks and their correlation). In other words, the sourcing of capital goods is a first-order determinant of optimal policy, and reshapes the way in which the central bank should react to sectoral shocks.

This paper relates to the work on monetary policy in production networks. On the positive side, a large literature shows that input–output linkages shape the transmission of monetary policy (Bouakez et al., 2009; Galesi and Rachedi, 2019; Pasten et al., 2020; Ghassibe, 2021; Afrouzi and Bhattarai, 2025). On the normative side, the production network alters the optimal response to sectoral disturbances, as the central bank should assign larger weights to upstream sectors (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). We complement this work by studying a different network, which captures transactions in capital goods across sectors. As with the input–output matrix, the investment network makes the transmission of monetary policy heterogeneous across sectors. Yet, because the interest rate is itself part of the user cost of capital, it reverses the optimal policy prescriptions of the input–output network.[1,2]

We also add to the literature showing that capital formation linkages across industries shape how sectoral shocks aggregate into macroeconomic fluctuations (Foerster et al., 2011; Atalay, 2017; vom Lehn and Winberry, 2022; Rubbo, 2026) and long-run growth (Foerster et al., 2022; Casal and Caunedo, 2026; Gaggl et al., 2026). We build on this work and show that the investment network is also central to understanding the transmission of monetary policy.

2 Empirical Evidence

This section provides evidence that monetary policy propagates through the investment network. Specifically, we ask whether a common monetary tightening produces different quantity and price responses in industries that sell investment goods, directly or indirectly, to the rest of the economy.

2.1 Data

We build a quarterly panel of real gross output, its price index, and investment upstreamness across 66 private U.S. industries, corresponding to three-digit NAICS codes, from

[1] Carlstrom and Fuerst (2005) and Sveen and Weinke (2005) study how the dependence of the user cost of capital on the interest rate alters the determinacy conditions of monetary-policy rules.

[2] This is reminiscent of the cost channel of monetary policy, which instead operates through a working-capital friction (Christiano et al., 2005; Ravenna and Walsh, 2006).

2005Q1 to 2026Q2. We use the GDP-by-Industry accounts of the U.S. Bureau of Economic Analysis (BEA), which provide data on real gross output and its price index for each industry. We complement them with the orthogonalized high-frequency monetary-policy surprises of Bauer and Swanson (2023), which we aggregate from the monthly to the quarterly frequency. While the original series ends in 2023Q4, we extend it through 2026Q2 following the procedure described in Appendix A.2.

We construct the empirical investment network for 1997 following the methodology of vom Lehn and Winberry (2022), adapted to our 66 industries. We combine sectoral investment expenditures by asset type from the BEA Fixed Assets Tables with the BEA Input–Output Tables. We remove the imported share of each capital commodity and assign the remaining domestic commodities to their producing industries using the BEA Make Table. Since the network predates the estimation sample, the position of an industry in the investment network is fixed with respect to the monetary policy shocks.

2.2 Econometric Specification

Our goal is not to measure how the investment network amplifies the aggregate effects of monetary policy, but rather to dissect the sectoral responses that lie behind them. To do so, we exploit the interaction between the time-series variation in monetary policy shocks and the cross-sectional variation in the position of industries in the investment network. We estimate these differential responses with the following panel local projection (Jordà, 2005), for each horizon $h = 0, \ldots, 12$:

$$Y^h_{s,t} = \mu^h_{s,q(t)} + \lambda^h_{b(s),t} + \beta_h m_t U^I_s + \gamma_h m_t U^H_s + \mathbf{X}'_{s,t-1}\theta_h + \varepsilon^h_{s,t}. \tag{1}$$

We consider two different dependent variables, $Y^h_{s,t}$. Our main one is the cumulative log change between $t-1$ and $t+h$ in real sectoral gross output. The second one is annualized quarterly inflation of the gross-output price index at horizon h.

Our key explanatory variable is the interaction of the monetary policy shock m_t with investment upstreamness U^I_s. Investment upstreamness measures the extent to which an industry supplies capital goods to the rest of the economy, both directly and through the successive rounds of capital-goods sales that its own customers generate. To measure it, we adapt the upstreamness index of Antràs et al. (2012) to capital-goods flows. Let $F^I_{x\to s}$ denote the domestic investment goods that industry x sells to industry s, let Z_x denote gross output of x, and let $\Psi^I = [\Psi^I_{xs}]$ collect the seller-by-customer shares $\Psi^I_{xs} \equiv F^I_{x\to s}/Z_x$, with suppliers in rows and customers in columns. Investment upstreamness is then

$$U^I_s \equiv \left\{(\mathsf{I} - \Psi^I)^{-1}\mathbf{1}\right\}_s = \left\{\left[\mathsf{I} + \Psi^I + (\Psi^I)^2 + \cdots\right]\mathbf{1}\right\}_s, \tag{2}$$

where I is the identity matrix and $\mathbf{1}$ is a vector of ones.

Investment upstreamness is highly concentrated, with half of the industries selling almost no capital goods. The investment hubs are, respectively, support activities for

mining, construction, machinery, and computer systems design.[3]

We also control for the interaction of the monetary policy shock with input–output upstreamness, $m_t U_s^H$. Let $F_{x\to s}^H$ denote the domestic intermediate inputs that industry x sells to industry s. We construct input–output upstreamness analogously from the 1997 BEA Use Table, using $\Psi^H = [\Psi_{xs}^H]$, where $\Psi_{xs}^H \equiv F_{x\to s}^H / Z_x$, and $U_s^H \equiv \{(\mathsf{I} - \Psi^H)^{-1}\mathbf{1}\}_s$. In this way, we ensure that the role of investment upstreamness in monetary transmission holds above and beyond that of the production network. The correlation between investment and input–output upstreamness is about -0.2, so their joint inclusion raises no collinearity concern.

The regression also includes a set of covariates, $\mathbf{X}_{s,t-1}$, which consist of four lags of the dependent and both explanatory variables. We further include industry-by-quarter-of-year fixed effects, $\mu_{s,q(t)}^h$, and 2-digit NAICS-sector-by-quarter fixed effects, $\lambda_{b(s),t}^h$, where $q(t)$ denotes the quarter of the year of period t and $b(s)$ maps each industry into its 2-digit NAICS sector. The former capture industry-specific seasonal patterns in activity and prices, while the latter absorb the monetary policy shock and other developments common to industries of the same broad sector in a given quarter.

Since our baseline regression features industry-by-quarter-of-year and NAICS-sector-by-quarter fixed effects, identification comes from variation in investment upstreamness across industries of the same NAICS sector. The NAICS-sector-by-quarter effects absorb the aggregate effect of monetary policy, whereas the industry-by-quarter-of-year effects absorb investment upstreamness, which does not vary over time. This setting allows us to identify how the transmission of a common monetary policy shock varies across industries depending on their position in the investment network.

The coefficient β_h measures the differential response per unit of investment upstreamness to a one-standard-deviation monetary policy shock. We report it multiplied by the standard deviation of U_s^I and normalized to a 25 bp shock. In other words, each reported coefficient is the difference in the response of two industries one standard deviation apart in investment upstreamness after a 25 bp monetary tightening.

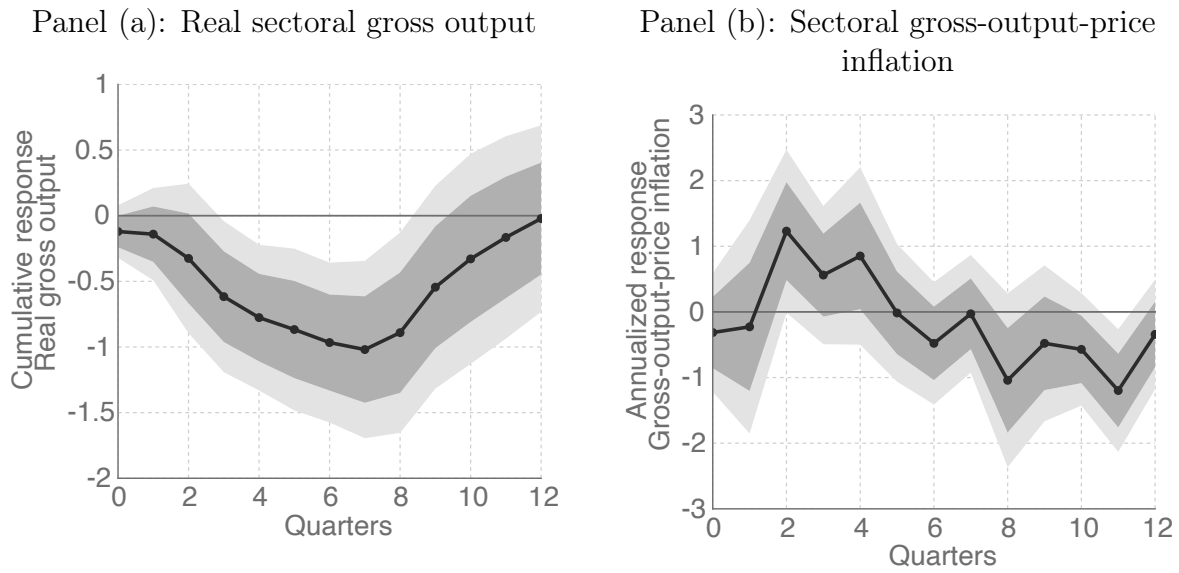
2.3 Results

We start by discussing the role of investment upstreamness in the transmission of monetary policy to real gross output. Panel (a) of Figure 1 shows its response with 68- and 90-percent Driscoll–Kraay confidence intervals.

Real gross output contracts more in investment-upstream industries following a monetary tightening. While the differential is close to zero on impact, it deepens over the following quarters, and is statistically significant at the 90-percent level from the third through the eighth quarter. At its peak, a one-standard-deviation increase in investment

[3]The position of each industry in the investment network is also remarkably stable over time: the correlation between investment upstreamness measured in 1997 and 2024 is 0.98.

Figure 1: Sectoral Responses by Investment Upstreamness.



Note: The figure reports the differential response to a one-standard-deviation increase in investment upstreamness after a 25 bp monetary tightening. Panel (a) plots cumulative real gross output; Panel (b) plots annualized quarterly gross-output-price inflation. The solid line with circles is the point estimate. The lighter and darker bands are the pointwise 90- and 68-percent Driscoll–Kraay confidence intervals.

upstreamness deepens the contraction by about 1.02 percentage points. To put this magnitude in context, the investment-upstreamness differential is about 90 percent as large as the input–output-upstreamness differential at their respective peaks. The data thus point to a first-order role for the investment network in the real transmission of monetary policy.

When we turn to inflation, Panel (b) of Figure 1 shows that inflation differentials are very limited and not statistically significant.

We validate these results in an extensive battery of robustness checks, reported in Appendix A. First, the same pattern holds when using real value added and value-added-price inflation rather than their gross-output counterparts. Second, we replace the baseline shock with that of Miranda-Agrippino and Ricco (2021), or end the sample in 2019Q4 to exclude COVID-related dynamics. Third, we change the fixed effects and the lag structure, or allow for industry-specific trends. Fourth, we consider alternative ways of computing standard errors: clustering by industry, two-way clustering by industry and quarter, and a moving-block bootstrap. Finally, we show that the role of the investment network holds above and beyond potential confounding factors. We do so in two ways: we add a third interaction between the monetary policy shock and either price rigidity or the Domar weight, and we add decile-by-quarter fixed effects for input–output upstreamness, price rigidity, and the Domar weight, so that the differential is identified within industries with similar characteristics.

Importantly, these results show that investment-upstream industries respond more to

monetary hikes, but do not by themselves identify how the response propagates through the network. Appendix A.8 investigates this mechanism in two ways. First, Appendix A.8.1 reports a spatial autoregression in the spirit of Ozdagli and Weber (2026) that relates the output response of each supplier to the responses of its investment customers. Second, in Appendix A.8.2, we follow the approach of Acemoglu et al. (2016) to separate an industry's direct sensitivity to monetary policy from its indirect exposure to its customers' investment demand. Both exercises uncover that monetary policy propagates through the investment network.

3 The Model

To quantify the propagation of monetary policy through the investment network, we build a multi-sector New Keynesian model with both a production and investment network. Specifically, the economy features S industries, denoted by $s \in \{1, \ldots, S\}$, which are interconnected by an input–output matrix, and are heterogeneous in demand composition, factor intensities, intermediate-input use, capital depreciation, and price rigidity. Each sector also operates a sector-specific stock of capital, which is accumulated out of a bundle of the goods supplied by all sectors, as specified by the investment network. Finally, the monetary authority sets the nominal interest rate with a Taylor rule.

We present the key parts of the model below; any additional detail is in Appendix B.

3.1 Household

The representative household chooses consumption C_t, labor N_t, one-period risk-free nominal bonds B_t, as well as sectoral investment $I_{s,t}$ and sectoral capital $K_{s,t+1}$ to maximize lifetime utility:

$$\max_{C_t, N_t, B_t, \{I_{s,t}, K_{s,t+1}\}_{s=1}^{S}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right) \right] \tag{3}$$

$$\text{s.t. } P_t^C C_t + \sum_{s=1}^{S} P_{s,t}^I I_{s,t} + B_t = W_t N_t + \sum_{s=1}^{S} R_{s,t}^K K_{s,t} + R_{t-1} B_{t-1} + \mathcal{D}_t, \tag{4}$$

where β is the discount factor, σ is risk aversion, θ is a labor disutility shifter, and η denotes the inverse of the Frisch elasticity of labor supply. Labor is perfectly mobile across sectors, so the household supplies a single quantity N_t at a single nominal wage W_t.

The budget constraint in Equation (4) posits that the household purchases consumption goods at price P_t^C and sectoral investment goods at price $P_{s,t}^I$. In addition, it saves in nominal bonds. The household finances expenditures with the proceeds of nominal bonds, whose gross nominal interest rate is R_t, nominal profits rebated from firms, $\mathcal{D}_t$, labor income at the nominal wage W_t, and capital income at the sectoral nominal returns $R_{s,t}^K$.

Physical capital is *sector specific*: the stock $K_{s,t}$ can be rented only to producers in

sector s. It accumulates with sectoral investment as:

$$K_{s,t+1} = (1-\delta_s)K_{s,t} + I_{s,t}\left[1 - \frac{\phi}{2}\left(\frac{I_{s,t}}{I_{s,t-1}} - 1\right)^2\right], \tag{5}$$

where δ_s is the sector-specific rate of depreciation, and ϕ captures the magnitude of the investment adjustment costs, which are common across sectors. These costs depend on the growth rate of investment, and vanish when investment is constant.

The fact that capital is sector-specific implies that each stock earns a different sectoral rental rate $R^K_{s,t}$, with an associated Tobin's-q per sector. The first-order condition for investment equates the price of the sectoral investment good to the value of the capital that an extra unit of it installs, net of adjustment costs:

$$\begin{aligned} P^I_{s,t} = Q^K_{s,t}\left[1 - \frac{\phi}{2}\left(\frac{I_{s,t}}{I_{s,t-1}} - 1\right)^2 - \phi\left(\frac{I_{s,t}}{I_{s,t-1}} - 1\right)\frac{I_{s,t}}{I_{s,t-1}}\right] + \dots \\ \cdots + \phi\,\mathbb{E}_t\left[\mathcal{M}_{t,t+1}Q^K_{s,t+1}\left(\frac{I_{s,t+1}}{I_{s,t}} - 1\right)\left(\frac{I_{s,t+1}}{I_{s,t}}\right)^2\right], \end{aligned} \tag{6}$$

where $Q^K_{s,t}$ denotes the shadow value of an additional unit of capital installed in sector s. The shadow value itself obeys the asset-value recursion:

$$Q^K_{s,t} = \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\left\{R^K_{s,t+1} + (1-\delta_s)\,Q^K_{s,t+1}\right\}\right], \tag{7}$$

where $\mathcal{M}_{t,t+1}$ is the household's nominal stochastic discount factor. An additional unit of installed capital pays the rental rate it earns next period and carries the continuation value of the part of it that does not depreciate.

Equations (6) and (7) deliver the user cost of capital of Jorgenson (1963), a margin through which monetary policy operates in this economy. A higher real interest rate discounts future rentals more heavily and depresses $Q^K_{s,t}$. As a result, sector s's investment demand falls, spilling over to its capital-goods suppliers.

3.2 Firms

Each sector consists of a continuum of monopolistically competitive producers, which supply differentiated varieties subject to Calvo pricing frictions; a representative wholesaler, which aggregates the varieties into a final sectoral good; and retailers, which assemble the sectoral goods into the consumption bundle, the investment bundle of each sector, and the intermediate-input bundle of each sector.

3.2.1 Producers

Within each sector, a continuum of monopolistically competitive producers indexed by $i \in [0,1]$ produces differentiated varieties of the sectoral good. Each producer combines production inputs to generate gross output with the Cobb–Douglas technology:

$$Z^i_{s,t} = \left(N^i_{s,t}\right)^{\alpha_{N,s}}\left(K^i_{s,t}\right)^{\alpha_{K,s}}\left(H^i_{s,t}\right)^{\alpha_{H,s}}, \tag{8}$$

where $Z^i_{s,t}$ is gross output of producer i in sector s at time t, and $N^i_{s,t}$, $K^i_{s,t}$, and $H^i_{s,t}$ denote production labor, capital, and intermediate inputs, respectively. The sector-specific parameters $\alpha_{N,s}$, $\alpha_{K,s}$, $\alpha_{H,s}$ denote the intensities of production labor, capital, and intermediates in gross output. We impose that $\alpha_{N,s} + \alpha_{K,s} + \alpha_{H,s} = 1$.

Producers purchase production labor and capital services from households at prices W_t and $R^K_{s,t}$, and intermediate inputs from retailers at price $P^H_{s,t}$. Each producer sets its nominal variety price $P^i_{s,t}$ subject to a price-setting friction à la Calvo (1983), such that producers can reset their price with the sector-specific probability $1 - \vartheta_s$. Thus, the parameter ϑ_s governs the intensity of nominal rigidities.

Specifically, given the household's nominal stochastic discount factor $\mathcal{M}_{t,\tau}$, producer i in sector s chooses the optimal reset price and the sequences of production inputs to maximize the expected discounted stream of future profits:

$$\max_{P^i_{s,t},\left\{N^i_{s,\tau},K^i_{s,\tau},H^i_{s,\tau}\right\}^{\infty}_{\tau=t}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \vartheta_s^{\tau-t} \mathcal{M}_{t,\tau} \left\{ P^i_{s,t} Z^i_{s,\tau} - W_\tau N^i_{s,\tau} - R^K_{s,\tau} K^i_{s,\tau} - P^H_{s,\tau} H^i_{s,\tau} \right\}, \quad (9)$$

subject to the technology (8) and the variety-level demand schedule stemming from the wholesalers' problem in the next section.

3.2.2 Wholesalers

In each sector, a perfectly competitive representative wholesaler purchases producers' varieties, $Z^i_{s,t}$, and bundles them into the sectoral good $Z_{s,t}$ with CES technology:

$$Z_{s,t} = \left[\int_0^1 Z^i_{s,t}{}^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}i \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (10)$$

where $\epsilon > 1$ is the elasticity of substitution across varieties within a sector. We denote by $P_{s,t}$ the sectoral price, and by $\Pi_{s,t} \equiv P_{s,t}/P_{s,t-1}$ gross sectoral inflation.

Wholesalers' goods are destined for consumption, investment, and intermediate-input use. The sectoral resource constraint reads:

$$Z_{s,t} = C_{s,t} + \sum_{x=1}^{S} I_{s\to x,t} + \sum_{x=1}^{S} H_{s\to x,t}, \quad (11)$$

where $C_{s,t}$ denotes the demand for the goods of sector s from the consumption retailer, $I_{s\to x,t}$ the demand for the goods of sector s from the retailer that assembles the investment good of sector x, and $H_{s\to x,t}$ the demand for the goods of sector s from the retailer that assembles the intermediate inputs used by producers in sector x. Throughout, the first subscript of a bilateral flow or weight denotes the supplier and the second the customer.

3.3 Retailers

Retailers assemble sectoral goods into three types of bundles: one for consumption and, for each sector, one for investment and one for intermediate inputs. The aggregators have the same CES form, but their weights and elasticities of substitution may differ.

3.3.1 The Consumption Retailer

A perfectly competitive representative consumption retailer bundles the sectoral consumption flows, $C_{s,t}$, into the consumption good, C_t, with the technology:

$$C_t = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_C}} C_{s,t}^{\frac{\nu_C - 1}{\nu_C}} \right]^{\frac{\nu_C}{\nu_C - 1}}, \tag{12}$$

where $\omega_{C,s}$ is the weight of the consumption goods of sector s, and ν_C is the elasticity of substitution across sectoral consumption goods. Conditional on ν_C and the reference relative prices, we choose the weights so that the retailer reproduces the contribution of each sector to personal consumption expenditures and the reference consumption price.

3.3.2 The Investment Retailers

Investment goods are assembled in the same fashion, the only difference being that there is one retailer for each sector. In each sector s, a perfectly competitive representative retailer assembles sectoral goods, $I_{x \to s,t}$, into the investment good used to accumulate the capital of sector s, $I_{s,t}$, with the technology:

$$I_{s,t} = \left[\sum_{x=1}^{S} \omega_{I,x \to s}^{\frac{1}{\nu_I}} I_{x \to s,t}^{\frac{\nu_I - 1}{\nu_I}} \right]^{\frac{\nu_I}{\nu_I - 1}}, \tag{13}$$

where $\omega_{I,x \to s}$ is the weight of goods supplied by sector x in the investment bundle of sector s, and ν_I is the elasticity of substitution across capital-goods suppliers. These weights define the *investment network*, which we collect in the matrix $\boldsymbol{\omega}_I \equiv [\omega_{I,s \to x}]$, with suppliers in rows and customers in columns.

3.3.3 The Intermediate-Input Retailers

Analogously, in each sector, a retailer assembles sectoral goods, $H_{x \to s,t}$, into the bundle of intermediate inputs used by sector s producers, $H_{s,t}$, with the technology:

$$H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,x \to s}^{\frac{1}{\nu_H}} H_{x \to s,t}^{\frac{\nu_H - 1}{\nu_H}} \right]^{\frac{\nu_H}{\nu_H - 1}}, \tag{14}$$

where $\omega_{H,x \to s}$ is the weight of sector x in the intermediate-input bundle of customer s, and ν_H is the elasticity of substitution across intermediate-input suppliers; these weights form the entries of the input–output matrix, $\boldsymbol{\omega}_H \equiv [\omega_{H,x \to s}]$, oriented like $\boldsymbol{\omega}_I$.

3.4 Monetary Authority

The monetary authority sets the nominal interest rate R_t with a standard Taylor rule that reacts to consumption-price inflation, $\Pi_t = P_t^C / P_{t-1}^C$, and the output gap, defined as the ratio between aggregate real GDP, Y_t, and that in the flexible-price economy, Y_t^f:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\phi_r} \left[\Pi_t^{\phi_\pi} \left(\frac{Y_t}{Y_t^f} \right)^{\phi_y} \right]^{1 - \phi_r} \exp(\varepsilon_t), \tag{15}$$

where $\bar{R}$ is the level of the nominal interest rate at the steady state,[4] ϕ_r governs the degree of interest-rate inertia, and ϕ_π and ϕ_y denote the responsiveness to inflation and the output gap, respectively. The variable ε_t is a monetary policy shock.

3.5 Closing the Model

The labor market clears economy-wide, the bond is in zero net supply, and firm profits are rebated to the household. Sectoral nominal value added, $\mathcal{Y}_{s,t}$, is the value of gross output net of intermediate-input purchases,

$$\mathcal{Y}_{s,t} \;=\; P_{s,t}Z_{s,t} \;-\; P^H_{s,t}H_{s,t}. \tag{16}$$

Summing across sectors and using market clearing delivers nominal GDP, $\mathcal{Y}_t$:

$$\mathcal{Y}_t = \sum_{s=1}^{S} \mathcal{Y}_{s,t} = P^C_t C_t + \sum_{s=1}^{S} P^I_{s,t} I_{s,t}. \tag{17}$$

To define aggregate real GDP, we set prices at their steady-state values and consider the consumption price as the numeraire, so that:

$$Y_t = C_t + \sum_{s=1}^{S} \frac{\bar{P}^I_s}{\bar{P}^C} I_{s,t}. \tag{18}$$

4 Calibration

We model the economy as consisting of $S = 66$ industries, corresponding to the BEA industries of the empirical analysis. A period in the model is one month. We consider a zero inflation rate in the steady state.

The household discount factor $\beta = 0.9984$ targets a 2 percent annual real interest rate. As standard, the risk aversion coefficient is $\sigma = 2$. The parameter $\eta = 0.67$ implies a 1.5 Frisch elasticity, in line with the evidence of Erosa et al. (2016), slightly below the Frisch elasticity of 2 considered in La'O and Tahbaz-Salehi (2022). The labor disutility parameter θ is set to achieve a steady-state value for aggregate labor of $\bar{N} = 0.33$.

The sectoral depreciation rates are the ratio of 2010–2024 current-cost depreciation to lagged year-end current-cost net capital from the BEA Fixed Assets Tables, converted into monthly frequency. The investment adjustment cost equals $\phi = 0.45$, to match the two-quarter peak response of investment to monetary policy shocks documented by Ottonello and Winberry (2020).

To set the elasticity of substitution of consumption, investment, and intermediate inputs across sectors, we follow the empirical evidence on these parameters. First, the elasticity of substitution of consumption across sectors is set to $\nu_C = 0.8$, in line with the estimates of Herrendorf et al. (2013). Second, we set the elasticity of substitution of investment across sectors to the same value, $\nu_I = 0.8$. Third, the elasticity of substitution of intermediate inputs across sectors is $\nu_H = 0.1$, following Atalay (2017), who highlights

[4] Throughout the paper, we refer to $\bar{X}$ as the steady-state value of variable X_t.

a stark complementarity of materials across industries in the short run.

Given the elasticities of substitution, the parameters $\omega_{C,x}$, $\omega_{I,x\to s}$, and $\omega_{H,x\to s}$ determine the heterogeneity in the sectoral contributions to consumption, the investment of every industry, and the use of intermediate inputs supplied by all industries, respectively. We discipline these parameters with the 2024 summary Use and Domestic Supply tables of the BEA: $\omega_{C,x}$ targets the sectoral shares in personal consumption expenditures, and $\omega_{H,x\to s}$ targets the sectoral shares in the supply and use of intermediate inputs with respect to all industries. To calibrate the investment network parameters, $\omega_{I,x\to s}$, we follow vom Lehn and Winberry (2022) by combining information on buyer's expenditures by asset from the 2024 BEA Fixed Assets Tables with the supplier composition of the private-fixed-investment columns of the 2024 Use Table.

We then set the elasticity of substitution across within-sector varieties to $\epsilon = 11$ to obtain a 10 percent markup. Given markups, the factor intensities $\alpha_{N,s}$, $\alpha_{K,s}$, and $\alpha_{H,s}$ target the sectoral shares of the compensation of employees, of gross operating surplus, and of the expenditures in intermediate inputs in gross output, which we measure with the 2024 Use Table. We set the sectoral Calvo probabilities, ϑ_s, to match the sectoral price durations of Pasten et al. (2020).

With respect to the monetary rule, we choose an interest-rate inertia of $\phi_r = 0.92832$, which implies a value of 0.8 at quarterly frequency, a response to inflation of $\phi_\pi = 1.5$, and a response to the output gap of $\phi_y = 0.125/12$.

5 Monetary Transmission

This section studies how the investment network shapes the transmission of monetary policy into sectoral activity and inflation. We proceed in two steps. Section 5.1 derives analytical results in a simplified version of the model, which transparently shows the channels at work. Section 5.2 then turns to the calibrated economy and quantifies the contribution of the investment network in accounting for the sectoral differential responses to monetary policy shocks.

5.1 Analytical Results

This section analytically characterizes sectoral monetary transmission by showing how a sector's position in the investment network shapes its value-added response to monetary policy shocks. We do so in a stripped-down economy in which the investment network is the *only* source of sectoral heterogeneity: industries differ solely in their position in the investment network, that is, in how much each of them supplies to the investment bundles of the others.

Section 5.1.1 presents the tractable economy, Section 5.1.2 characterizes the investment network, Section 5.1.3 derives the log-linearized equilibrium, and Section 5.1.4 characterizes the cumulative response of sectoral value added. All proofs are relegated to Appendix C.

5.1.1 The Tractable Economy

The analytical economy is obtained from the full model by imposing these restrictions:

(i) *Logarithmic consumption and linear labor*: $\sigma = 1$ and $\eta = 0$, so that period utility is $U(C_t, N_t) = \log C_t - \theta N_t$.

(ii) *No intermediate inputs*: $\alpha_{H,s} = 0$ for every s, so that output is produced only with labor and capital, and the input–output matrix plays no role.

(iii) *Homogeneous factor intensities*: the labor intensity is $\alpha \in (0,1)$, and the capital intensity is $1-\alpha$, common to all sectors.

(iv) *Homogeneous price rigidity*: $\vartheta_s = \vartheta \in (0,1)$ for every s, so that every industry faces the same slope of the Phillips curve, $\kappa \equiv (1-\vartheta)(1-\beta\vartheta)/\vartheta$.

(v) *Homogeneous consumption contribution*: $\omega_{C,s} = 1/S$ for every s, so that every industry contributes equally to the household's consumption expenditure.

(vi) *Homogeneous depreciation*: $\delta_s = \delta \in (0,1)$ for every s.

(vii) *Frictionless, contemporaneous capital*: $\phi = 0$, so that there are no investment adjustment costs, and capital installed at t is productive at t, so that the law of motion of capital in Equation (5) becomes $K_{s,t} = (1-\delta)K_{s,t-1} + I_{s,t}$.

(viii) *Cobb–Douglas final-demand bundles*: $\nu_C = \nu_I = 1$, so that consumption and sectoral investment expenditure shares are constant.

(ix) *Efficient steady state*: producers receive a constant sales subsidy $\varsigma = 1/(\epsilon - 1)$ on revenue, financed by a lump-sum tax on the household, which offsets the steady-state markup so that marginal cost equals the price in every sector, $\bar{\Xi}_s = \bar{P}_s$; Appendix B.2 writes the price-setting condition under the subsidy.

The investment supplier-by-customer matrix $\boldsymbol{\omega}_I = [\omega_{I,s\to x}]$ is therefore the only primitive source of sectoral heterogeneity. Its element $\omega_{I,s\to x}$ is the weight of investment goods sold by sector s to sector x. We refer to the seller as the supplier and to the buyer as the customer. With unit elasticity of substitution, this weight equals the share of sector x's investment expenditure allocated to goods supplied by sector s. The weights sum to one for each customer, $\sum_{s=1}^{S} \omega_{I,s\to x} = 1$, so $\boldsymbol{\omega}_I$ is column-stochastic.

Throughout, we set the price of the consumption basket, P_t^C, as the numeraire of the economy, which pins down all relative prices in the economy. Specifically, we denote by $q_{s,t} = P_{s,t}/P_t^C$ the relative price of the good of sector s; $q_{s,t}^I = P_{s,t}^I/P_t^C$ is the relative price of the investment good of sector s; $q_{s,t}^K = Q_{s,t}^K/P_t^C$ is the real shadow value of installed capital in sector s. The real wage is $w_t = W_t/P_t^C$. Then, $r_{s,t}^K = R_{s,t}^K/P_t^C$ denotes the real rental rate of capital in sector s, and $r_t = R_t/\mathbb{E}_t\Pi_{t+1}$ is the ex-ante real rate.

To fix notation, we denote by $\hat{X}_{s,t} \equiv \log(X_{s,t}/\bar{X}_s)$ the log deviation of any variable $X_{s,t}$ from its steady state. Bold letters collect the corresponding sectoral vectors and

matrices, as in $\hat{\mathbf{X}}_t \equiv (\hat{X}_{1,t}, \ldots, \hat{X}_{S,t})'$. The symbols $\mathbf{1}$ and $\mathbf{0}$ denote vectors of ones and zeros, I is the identity matrix, and $\mathbf{e}_s$ is its s-th column.

5.1.2 The Investment Network

This section characterizes how the investment network shapes the sales of each sector. First, the share of investment expenditure of each sector relative to its own sales, which is common across sectors, is given by

$$\gamma \equiv \frac{(1-\alpha)\delta}{1-\beta(1-\delta)}. \tag{19}$$

The numerator captures the flow of capital purchases per unit of sales: the capital share $1-\alpha$ pins down how much capital the sector uses per unit of sales, and the depreciation rate δ pins down how much of that capital it replaces each period. The denominator is the steady-state value of the rental rate of capital.

Second, let $v_s = \bar{q}_s \bar{Z}_s / \bar{C}$ denote the steady-state sales of sector s relative to aggregate consumption. Sector s sells to households, which spend a share $\omega_{C,s}$ of their consumption on its goods, and its investment customers, each of which spends a share γ of its own sales on capital goods and purchases a fraction $\omega_{I,s\to x}$ of them from sector s. Thus,

$$v_s = \omega_{C,s} + \gamma \sum_{x=1}^{S} \omega_{I,s\to x}\, v_x. \tag{20}$$

Since the sales of each customer depend in turn on the investment of its own customers, v_s is pinned down by the investment network. We refer to v_s as the *social value* of sector s, because Section 6 shows that it coincides with the weight that the planner assigns to sector s in the welfare loss. Since consumption shares are homogeneous, any sectoral dispersion in social value comes from the investment network. The social value is therefore larger in *investment hubs*.

Third, the sales of sector s split between the two sources of demand, that is, consumption and investment. The share that goes to consumption is $s_s^C \equiv \omega_{C,s}/v_s$, and the share that goes to the investment of customer x is

$$\psi_{s\to x} \equiv \frac{\gamma\, \omega_{I,s\to x}\, v_x}{v_s}. \tag{21}$$

Summing these shares over all customers gives the investment-sales intensity of sector s, $\ell_s \equiv \sum_{x=1}^{S} \psi_{s\to x}$. Since the two terms of Equation (20) exhaust the sales of sector s, the two shares add up to one,

$$s_s^C + \ell_s = 1. \tag{22}$$

Finally, we introduce the object that captures the position of each sector in the investment network, its *investment upstreamness*, which coincides with the measure of Equation (2) used in the empirical analysis. To do so, we first let Ψ^I collect the shares $\psi_{s\to x}$, with suppliers in rows and customers in columns. The first power of Ψ^I captures the direct orders that customers place with each supplier, while higher powers add the

indirect ones. Summing over all these rounds gives the Leontief inverse

$$\mathsf{L}_I \equiv (\mathsf{I} - \Psi^I)^{-1} = \sum_{k\geq 0} (\Psi^I)^k. \tag{23}$$

Then, investment upstreamness is the total investment demand that reaches sector s through all these rounds,

$$U_s^I \equiv \sum_{x=1}^{S} [\mathsf{L}_I]_{s\to x}. \tag{24}$$

5.1.3 Log-Linearized Equilibrium

To characterize the monetary transmission through the investment network, we first log-linearize the equilibrium conditions around the deterministic steady state, and then cumulate them over all horizons. Throughout the analytical exercises, we assume $\gamma < \delta/(2-\delta)$, which rules out explosive capital dynamics for any investment network. Under this condition, capital returns to its steady state after a transitory monetary shock, as shown in Appendix C.3.

Consumption and the real wage. The household Euler equation is

$$\hat{C}_t \;=\; \mathbb{E}_t \hat{C}_{t+1} - \hat{r}_t. \tag{25}$$

A monetary tightening lowers consumption by raising the real return to saving. Iterating the Euler equation forward, with consumption returning to its steady state, gives

$$\hat{C}_t = -\sum_{q\geq 0} \hat{r}_{t+q}, \tag{26}$$

so that consumption falls with the entire path of current and future real interest rates. As in Hansen (1985) and Rogerson (1988), linear labor disutility delivers a perfectly elastic labor supply, so that the real wage in consumption units moves one-for-one with consumption:

$$\hat{w}_t \;=\; \hat{C}_t. \tag{27}$$

Relative prices. In this setting, monetary policy shocks do not move relative prices. On the one hand, perfect labor mobility implies that all sectors pay the same wage. On the other hand, since capital installed today is already productive today, sectors adjust their capital stock rather than their rental rate when their demand falls, so that the rental rate is common across sectors. Thus, sectoral relative prices stay at their steady-state values, $\hat{q}_{s,t} = 0$, as do investment prices, $\hat{q}^I_{s,t} = 0$, for every s.

Supplier demand. Since relative prices do not move, the output of each supplier responds to consumption and the investment of its customers, in proportion to the shares s^C_s and $\psi_{s\to x}$. Thus, the goods-market clearing at the sectoral level is

$$\hat{Z}_{s,t} \;=\; s^C_s\, \hat{C}_t + \sum_{x=1}^{S} \psi_{s\to x}\, \hat{I}_{x,t}. \tag{28}$$

Capital demand. The capital that each sector uses per unit of output depends only on

the wage relative to the rental rate of capital. We derive capital demand by combining the first-order conditions of producers for labor and capital, and substituting the wage with consumption by Equation (27):

$$\hat{K}_{s,t} - \hat{Z}_{s,t} = \alpha\big(\hat{C}_t - \hat{r}^K_{s,t}\big). \tag{29}$$

Intuitively, a sector uses less capital per unit of output when (i) consumption falls, since labor becomes cheaper; and (ii) the rental rate rises, since capital becomes more expensive. The capital stock accumulates from investment,

$$\hat{K}_{s,t} = (1-\delta)\hat{K}_{s,t-1} + \delta\,\hat{I}_{s,t}, \tag{30}$$

so that the cumulative responses of capital and investment coincide.

The user cost of capital. Since relative prices do not move, the price of a newly installed unit of capital stays at its steady-state value in every sector. Thus, the capital Euler equation delivers the user cost of Jorgenson (1963), in which the real rate raises the rental rate by the same amount in every sector,

$$\hat{r}^K_{s,t} \;=\; \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\,\hat{r}_t. \tag{31}$$

Value added. Without intermediate inputs, real sectoral value added coincides with gross output,

$$\hat{Y}_{s,t} = \hat{Z}_{s,t}, \tag{32}$$

where $Y_{s,t}$ denotes sectoral real value added, defined in Equation (B.26) of Appendix B.5, so that Equation (28) also delivers the response of value added.

Cumulating the equilibrium. Let $\mathcal{Z}_s \equiv \sum_{t\geq 0}\hat{Z}_{s,t}$, $\mathcal{C} \equiv \sum_{t\geq 0}\hat{C}_t$ and $\mathcal{R} \equiv \sum_{t\geq 0}\hat{r}_t$ denote cumulative responses along the impulse response.[5] Summing Equation (26) over all horizons gives cumulative consumption as a function of the path of the real interest rate alone,

$$\mathcal{C} = -\sum_{t\geq 0}(t+1)\,\hat{r}_t. \tag{33}$$

Throughout this section, we define a monetary tightening as a path of the real interest rate that lies above its steady state: $\hat{r}_t \geq 0$ at every horizon, with $\hat{r}_0 > 0$. We take this path as given, so that the results below hold whatever the monetary rule that implements it.[6] A tightening then delivers $\mathcal{R} > 0$ and, by Equation (33), $\mathcal{C} < 0$. Equation (26) further implies that consumption does not exceed its steady state at any horizon, with $\hat{C}_0 = -\mathcal{R}$.

[5] To ensure that the cumulative responses are finite, we consider real interest-rate paths such that $\sum_{t\geq 0}(t+1)|\hat{r}_t| < \infty$.

[6] For a given path of the real interest rate, the Phillips curve pins down inflation, and the nominal interest rate follows from the Fisher equation. Under the Taylor rule of Section 3.4, the path of the real rate is instead endogenous. Since capital installed today is already productive today, the real rate enters current marginal cost through the rental rate of Equation (31), with a coefficient $(1-\alpha)\beta(1-\delta)/[1-\beta(1-\delta)]$ that is large when capital is durable, against a coefficient α on consumption. A higher real rate can therefore raise inflation, and a positive innovation to the rule can lower the real rate on impact. Defining the tightening in terms of the real rate avoids this ambiguity.

5.1.4 The Response of Sectoral Value Added

This section characterizes the cumulative response of sectoral real value added to a monetary policy shock. We first build intuition in a two-sector example, and then generalize the results to a general investment network.

A two-sector example. To build intuition before stating the proposition, consider a stylized two-sector economy in which sector 1 is fully upstream and sector 2 fully downstream:

$$\omega_{I,1\to1} = \omega_{I,1\to2} = 1, \qquad \omega_{I,2\to1} = \omega_{I,2\to2} = 0. \tag{34}$$

Both sectors purchase investment goods from sector 1, while neither purchases them from sector 2. Equations (20)–(21) imply

$$v_1 = \frac{1+\gamma}{2(1-\gamma)}, \qquad v_2 = \frac{1}{2}, \qquad \Psi^I = \begin{pmatrix} \psi_{1\to1} & \psi_{1\to2} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \gamma & \dfrac{\gamma(1-\gamma)}{1+\gamma} \\ 0 & 0 \end{pmatrix}. \tag{35}$$

Thus, sector 1 is the investment hub of this economy. These shares imply the following upstreamness measures for each sector:

$$U_1^I = 1 + \frac{\psi_{1\to1} + \psi_{1\to2}}{1-\psi_{1\to1}} > 1, \qquad U_2^I = 1. \tag{36}$$

To see how the demand of the two sectors reaches sector 1, we cumulate Equations (28), (29) and (31) over all horizons, and use the fact that the cumulative responses of capital and investment coincide:

$$\mathcal{Z}_1 = s_1^C \mathcal{C} + \psi_{1\to1}\left\{\mathcal{Z}_1 + \alpha\left[\mathcal{C} - \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\mathcal{R}\right]\right\} + \psi_{1\to2}\left\{\mathcal{Z}_2 + \alpha\left[\mathcal{C} - \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\mathcal{R}\right]\right\}, \tag{37}$$

$$\mathcal{Z}_2 = \mathcal{C}. \tag{38}$$

The sales of sector 1 come from three sources: the consumption of households, in proportion to s_1^C, the share of its sales that goes to consumption; its own investment, in proportion to $\psi_{1\to1}$, the share that goes to its own investment; and the investment of sector 2, in proportion to $\psi_{1\to2}$, the share that goes to the investment of sector 2. The investment of each sector moves with its own output and with consumption, and falls when the rental rate of capital rises. Sector 2, instead, provides no investment goods, so that its sales come only from households' consumption.

Solving for the sales of the two sectors yields

$$\mathcal{Z}_1 = \underbrace{\left[1 + \alpha\left(U_1^I - 1\right)\right]\mathcal{C}}_{\text{final-demand effect}} - \underbrace{\alpha\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\left(U_1^I - 1\right)\mathcal{R}}_{\text{user-cost effect}}, \tag{39}$$

$$\mathcal{Z}_2 = \underbrace{\mathcal{C}}_{\text{final-demand effect}}. \tag{40}$$

The first term captures the *final-demand effect*. A monetary tightening lowers consumption and, with it, the real wage. As a result, both sectors use less capital per unit of output,

and cut their investment orders. The second term captures the *user-cost effect*: the rental rate of capital rises with the real interest rate, which leads both sectors to cut their investment orders further. Hence, the investment network leads sector 1 to experience a larger drop in sales than sector 2. Crucially, the size of this drop is modulated by investment upstreamness, U_1^I. Should capital fully depreciate, then the user-cost effect would disappear.

In a general investment network, the demand of each customer reaches its suppliers, the suppliers of these suppliers, and so on. The Leontief inverse behind investment upstreamness captures all these effects, as the next proposition shows.

Proposition 1 (Cumulative response and investment upstreamness)**.** *After a monetary tightening, the cumulative response of the real value added of sector x is*

$$\mathcal{Z}_x \;=\; \underbrace{\big[1+\alpha\big(U_x^I-1\big)\big]\mathcal{C}}_{\text{final-demand effect}} \;-\; \underbrace{\alpha\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\big(U_x^I-1\big)\,\mathcal{R}}_{\text{user-cost effect}}. \tag{41}$$

Both effects are modulated by the investment network via the upstreamness measure.

Proof. See Appendix C.3. □

Since both effects depend on the investment network only through investment upstreamness, the response of a sector relative to another depends only on their positions in the network. The next corollary shows that the contraction of value added is stronger in the sector that is more investment-upstream. When all sectors share the same position, as in a homogeneous investment network, this differential disappears.

Corollary 1 (Sectoral value added and investment upstreamness)**.** *After a monetary tightening:*

(i) For any pair of suppliers x and y, the cumulative value-added differential is

$$\mathcal{Z}_x-\mathcal{Z}_y=\alpha\big(U_x^I-U_y^I\big)\Big[\mathcal{C}-\frac{\beta(1-\delta)}{1-\beta(1-\delta)}\mathcal{R}\Big]. \tag{42}$$

(ii) Since a tightening delivers $\mathcal{C}<0$ and $\mathcal{R}>0$, the term in brackets in Equation (42) is negative, so that

$$U_x^I>U_y^I \quad\Longrightarrow\quad \mathcal{Z}_x<\mathcal{Z}_y. \tag{43}$$

The contraction of real value added is therefore stronger the more investment-upstream a sector is.

(iii) If $\boldsymbol{\omega}_I=S^{-1}\mathbf{1}\mathbf{1}'$, then $\mathbf{U}^I=(1-\gamma)^{-1}\mathbf{1}$ and all sectoral responses are identical. The homogeneous investment network produces no upstream–downstream differential.

Proof. See Appendix C.4. □

Thus, the real effects of monetary policy are stronger in investment-upstream industries. Inflation, instead, carries no differential across sectors. Since relative prices do not move,

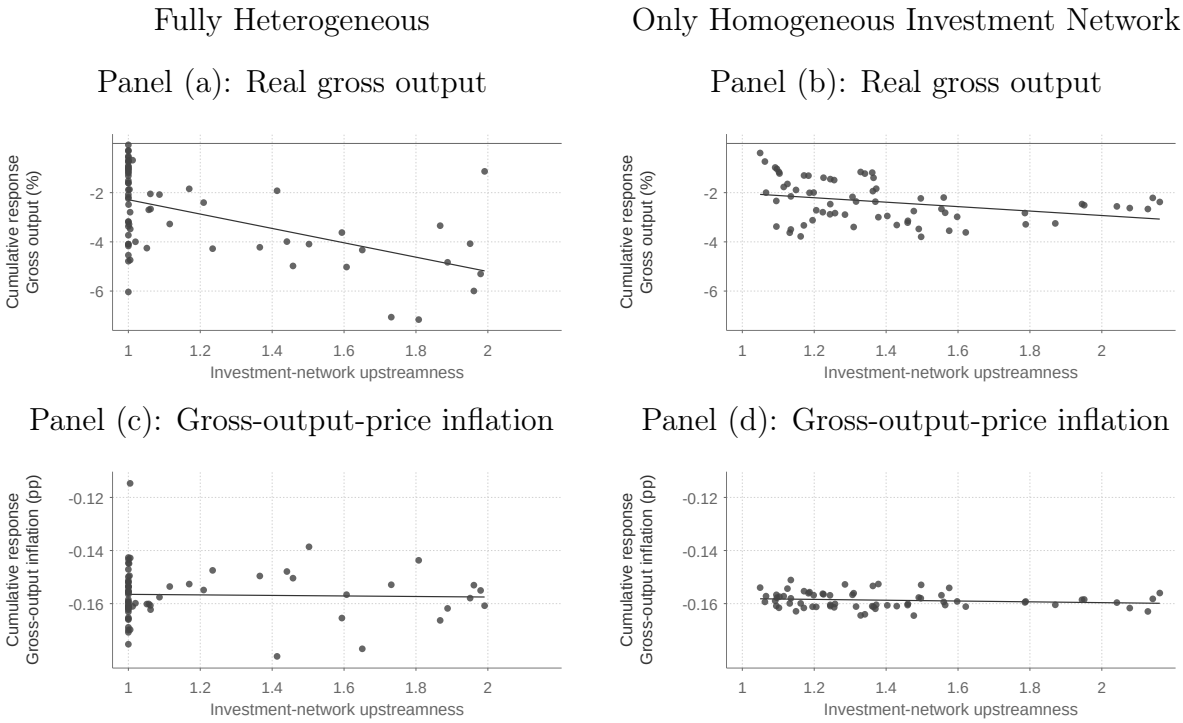
all sectors set the same price path, so that sectoral inflation coincides with aggregate inflation.

Importantly, in this analytical economy the heterogeneity of the investment network alters the differences across sectors, but not the aggregate: the responses of aggregate GDP and the real rate do not depend on the structure of the investment network.

5.2 Quantitative Analysis

We now use the calibrated model to quantify the role of the investment network in sectoral monetary transmission and compare its predictions with the data. As in our empirical analysis, we consider a 25 bp monetary tightening and study the cumulative log change in sectoral gross output and inflation over the first two years, which is roughly when the response peaks in the data. We conduct this exercise in the fully heterogeneous model and in a counterfactual economy in which only the investment network is set homogeneously across industries. This comparison allows us to assess the quantitative performance of the model and isolate the contribution of the investment network.

Figure 2: Sectoral Responses and the Investment Network.



Note: The figure compares the fully heterogeneous economy (left column) with the economy in which only the investment network is set homogeneously across industries (right column). Panels (a) and (b) report real gross output, and Panels (c) and (d) report gross-output-price inflation. The 2-year cumulative responses to a 25 bp monetary tightening are plotted against each economy's investment upstreamness. Dots represent the 66 industries and solid lines are linear fits.

Panel (a) of Figure 2 shows that real gross output contracts substantially more in investment-upstream industries. Specifically, along the fitted line, a one-standard-deviation increase in investment upstreamness deepens the contraction by 0.92 percentage points,

nearly matching the empirical estimate of 1.02 percentage points. To put this magnitude in context, the additional contraction amounts to about 30 percent of the average response across industries. Panel (b) shows that in the economy with the homogeneous investment network there is no quantitatively relevant relationship between investment upstreamness and the contraction in gross output across industries. In other words, heterogeneity in capital flows across industries crucially shapes the sectoral output responses to monetary policy.

Panels (c) and (d) report the corresponding responses of gross-output-price inflation. In both the fully heterogeneous economy and the economy with a homogeneous investment network, inflation is essentially unrelated to investment upstreamness. This is somewhat expected: upstreamness records what an industry sells into capital formation, whereas the policy rate reaches its price through the capital the industry itself rents and the prices its suppliers charge.[7] They vary instead with each industry's interest-rate exposure in marginal cost, $\alpha_{K,s}\,\beta(1-\delta_s)/[1-\beta(1-\delta_s)]$, cumulated through the input–output and investment networks. Consistent with the data, investment-upstream industries therefore experience larger contractions in gross output without systematically different inflation responses.

These exercises show that the model can account, both qualitatively and quantitatively, for how the sectoral responses of gross output and inflation vary with industries' position in the investment network. As such, it provides an ideal laboratory for measuring the contribution of the investment network to the transmission of monetary policy.

We then use the model to quantify the investment network's contribution to aggregate monetary transmission. While sectoral heterogeneity is known to amplify the real effects of monetary policy (Pasten et al., 2020), we ask how much of this amplification comes from the investment network. We measure its share of the overall amplification in a response x_h at horizon h as

$$\frac{x_h^{\text{het}} - x_h^{\text{no inv}}}{x_h^{\text{het}} - x_h^{\text{hom}}},$$

where "het" denotes the fully heterogeneous calibration, "no inv" the economy in which we shut down heterogeneity only in the investment network, and "hom" the fully homogeneous economy. The denominator measures the overall amplification due to sectoral heterogeneity, while the numerator measures how much of it disappears when we shut down heterogeneity in the investment network, keeping all other sectoral characteristics unchanged.

Table 1 reports the responses of these three economies to the same monetary tightening. After one quarter, the cumulative GDP response is -0.73% in the fully heterogeneous economy, compared with -0.57% in the fully homogeneous economy and -0.62% when only the investment network is homogeneous. After one year, the corresponding responses

[7] In fact, under restrictions (iii) and (vi) of Section 5.1.1 this exposure is common across industries, so sectoral inflation responses are identical whatever the shape of the investment network; Panel (c) departs from that benchmark only through heterogeneity in $\alpha_{K,s}$ and δ_s.

Table 1: Aggregate Amplification and the Investment Network.

	GDP		Inflation	
Economy	One quarter	One year	One quarter	One year
Fully homogeneous	−0.57	−0.94	−0.14	−0.17
Only investment network homogeneous	−0.62	−1.18	−0.13	−0.16
Fully heterogeneous	−0.73	−1.59	−0.13	−0.16
Investment-network share of amplification	70%	63%	0%	0%

Note: The first three rows report the cumulative one-quarter and one-year responses of GDP and inflation to a monetary tightening in three model specifications: fully homogeneous, homogeneous only in the investment network, and fully heterogeneous. The last row reports the investment network's share of the response difference between the fully heterogeneous and fully homogeneous economies.

are −1.59%, −0.94%, and −1.18%. These differences imply that the investment network accounts for 70 percent of the additional GDP contraction after one quarter and 63 percent after one year.

For inflation, the investment network generates no amplification whatsoever. This implies that the ratio of the absolute GDP response to the absolute inflation response rises from 4.8 to 5.6 after one quarter, and from 7.4 to 9.9 after one year, when we restore heterogeneity in the investment network. Thus, the investment network substantially amplifies the real effects of monetary policy shocks.

6 Optimal Monetary Policy

This section studies how the investment network alters optimal monetary policy. Section 6.1 characterizes optimal policy analytically in the simplified framework of Section 5.1. Section 6.2 then quantifies it in the calibrated economy.

6.1 Analytical Results

We consider the same economy as in Section 5.1, and derive the optimal monetary policy under commitment, in which the monetary authority chooses the complete state-contingent path of the interest rate. This is done under the premise that the steady state is efficient due to the presence of a sales subsidy. We present the key steps below; additional details and derivations are in Appendix D.

We proceed in four steps. We first introduce sectoral cost-push shocks, which move relative prices, and thus sectoral inflation. We then derive the welfare losses of households. Next, we show that the problem of the planner collapses to an aggregate one, in which the investment network enters through the social value of each sector. Finally, we characterize the optimal path of the interest rate, and how it depends on the position in the investment network of the sector that is hit by a cost-push shock.

Cost-push shocks. We replace the monetary policy shock with transitory sectoral cost-push shocks, $u_{s,t}$, which enter the Phillips curve of each sector,

$$\pi_{s,t} = \beta \mathbb{E}_t \pi_{s,t+1} + \kappa \hat{\Xi}_{s,t} + u_{s,t}, \tag{44}$$

where κ is the common slope. The marginal cost of each sector relative to its price, $\hat{\Xi}_{s,t}$, features three components,

$$\hat{\Xi}_{s,t} = \hat{C}_t + \frac{1-\alpha}{\alpha}\left(\hat{Z}_{s,t} - \hat{K}_{s,t}\right) - \hat{q}_{s,t}. \tag{45}$$

First, the real wage moves one-for-one with consumption, so that marginal cost rises with consumption. Second, producing more output with a given stock of capital requires more labor per unit of capital, which lowers the marginal product of labor. Accordingly, the marginal cost rises with output per unit of capital. Third, since the wage is measured in units of the consumption basket, whereas the marginal cost is measured relative to the price of the sector, marginal cost falls when the relative price of the sector rises.

Unlike in Section 5.1, the presence of the sector-specific cost-push shocks implies that now relative prices do move, and so do marginal costs. As a result, a cost-push shock generates an inflation differential for the shocked sector.[8] A single interest rate cannot stabilize the inflation of all sectors at once, so that the planner faces a genuine tradeoff.

Welfare. A second-order approximation of the welfare of households yields the following period loss:

$$\begin{aligned}\mathcal{L}_t = \underbrace{\hat{C}_t^2}_{\substack{\text{consumption}\\ \text{gap}}} + \underbrace{\frac{\epsilon}{\kappa}\sum_{s=1}^{S} v_s\, \pi_{s,t}^2}_{\text{price dispersion}} + \underbrace{\alpha(1-\alpha)\sum_{s=1}^{S} v_s\left(\hat{N}_{s,t} - \hat{K}_{s,t}\right)^2}_{\text{factor mix}} + \dots \\ \dots + \underbrace{\sum_{x=1}^{S} \omega_{C,x}\, \hat{q}_{x,t}^2}_{\text{consumption mix}} + \underbrace{\gamma \sum_{s=1}^{S} v_s \sum_{x=1}^{S} \omega_{I,x\to s}\left(\hat{q}_{x,t} - \hat{q}_{s,t}^I\right)^2}_{\text{investment mix}}.\end{aligned} \tag{46}$$

The welfare loss features five terms. The first, the consumption gap, captures the curvature of utility: fluctuations of consumption lower welfare. The second, price dispersion, captures the resources that the dispersion of prices across the producers of each sector wastes. The third, the factor mix, captures the resources that an inefficient labor–capital ratio wastes in production, with $N_{s,t}$ denoting the labor employed in sector s. The fourth and the fifth, the consumption mix and the investment mix, capture the fact that movements in relative prices induce substitution towards cheaper goods, so that the consumption and the investment bundles depart from their efficient composition.

The investment network shapes the welfare loss, and it does so through the sectoral

[8] Relative prices also enter two of the remaining equilibrium conditions of Section 5.1.3. Goods-market clearing reflects the reallocation of demand across sectors that relative prices induce, and the user cost of capital moves with the price of investment goods, which is a weighted average of the relative prices of the suppliers.

social value, v_s. The sales of each sector include the investment demand of its customers, so that the social value is larger in investment hubs. It modulates three terms: the price dispersion, the factor mix, and the investment mix. These three losses scale with the sales of the sector, so that a distortion is more costly in a sector with a larger social value.

The planner's problem. The interest rate moves the marginal cost of all sectors in the same way, so that monetary policy cannot alter relative prices. Hence, the planner only controls aggregate variables. This, in turn, determines the terms of the welfare loss that the planner can actively affect. While the planner can move the consumption gap, aggregate consumption inflation, π_t, and the factor mix via changes in the interest rate,[9] it cannot move the inflation of a sector relative to the others, nor the consumption and investment mix, which depend on relative prices alone.

Let $V \equiv \sum_{s=1}^{S} v_s$ denote aggregate social value. Up to terms independent of policy, the planner chooses the paths of consumption, aggregate inflation, and the real interest rate to minimize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \underbrace{\hat{C}_t^2}_{\substack{\text{consumption} \\ \text{gap}}} \underbrace{+ \frac{\epsilon}{\kappa} V \pi_t^2 + 2 \frac{\epsilon}{\kappa} \pi_t \sum_{s=1}^{S} v_s (\hat{q}_{s,t} - \hat{q}_{s,t-1})}_{\text{price dispersion}} + \dots \\ \dots + \underbrace{\alpha(1-\alpha) V \left[\frac{1}{V} \sum_{s=1}^{S} v_s \hat{r}_{s,t}^K - \hat{C}_t \right]^2}_{\text{factor mix}} \Bigg\}. \tag{47}$$

The first term is the consumption gap, as before. The second term captures the resources wasted by price dispersion. It depends on aggregate inflation as well as on the changes in sectoral relative prices, weighted by social value. This is because aggregate inflation is more costly when it comes from the surge of the relative price of a high-social-value sector.

The third term is the factor mix. The expression in brackets is the social-value-weighted average rental rate of capital minus the real wage. In what follows, we denote this factor-mix gap by G_t:

$$G_t \equiv \frac{1}{V} \sum_{s=1}^{S} v_s \hat{r}_{s,t}^K - \hat{C}_t. \tag{48}$$

As implied by the investment and capital optimality conditions of the full model, Equations (6) and (7), the rental rate of capital moves with three forces. First, a higher current relative price of investment goods raises it. Second, when capital is durable, a higher real interest rate raises it. Third, expected growth in the relative price of investment goods lowers it through capital gains. Consumption appears in this gap as it pins down the real

[9] The labor–capital ratio of each sector depends on its rental rate relative to the wage. The interest rate moves the rental rate of every sector through the user cost of capital, and consumption moves the wage, so that the planner can align the average user cost of capital with the real wage. The dispersion of the rental rate across sectors, which depends on relative prices, is instead beyond its reach.

wage. Monetary policy can therefore move the average gap, even though it cannot change the dispersion of rental rates across sectors.

The planner faces two constraints. The first is the consumption Euler equation,

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} - \hat{r}_t.$$

The second is the aggregate Phillips curve,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \Big[\alpha \hat{C}_t + (1-\alpha) \frac{\beta(1-\delta)}{1-\beta(1-\delta)} \hat{r}_t \Big] + f_t, \tag{49}$$

where the forcing term f_t collects the direct cost-push wedge and the investment-price component of marginal costs:

$$f_t = \sum_{s=1}^{S} \omega_{C,s} \, u_{s,t} + \kappa(1-\alpha) \sum_{s=1}^{S} \omega_{C,s} \, \frac{\hat{q}^I_{s,t} - \beta(1-\delta) \, \mathbb{E}_t \hat{q}^I_{s,t+1}}{1-\beta(1-\delta)}. \tag{50}$$

The first term raises inflation directly; the second changes marginal costs through the price of investment goods, and the user cost of capital.

Let $\hat{q}^I_t \equiv V^{-1} \sum_{s=1}^{S} v_s \hat{q}^I_{s,t}$ denote the average relative price of investment goods, weighted by social value. Proposition 2 characterizes the optimal path of the interest rate.

Proposition 2 (Optimal policy)**.** *The optimal interest rate under commitment satisfies*

$$\hat{r}^*_t = \underbrace{\frac{\alpha(1-\alpha) V \, \mathbb{E}_t \Delta \hat{q}^I_{t+1} - \alpha\epsilon \, \mathbb{E}_t \sum_{s=1}^{S} v_s \pi_{s,t+1}}{1+\alpha(1-\alpha)V}}_{\text{price stabilization}} + \underbrace{\frac{\frac{\beta(1-\delta)}{1-\beta(1-\delta)}(1-\alpha) \mathbb{E}_t \Delta \mathcal{B}_{t+1}}{1+\alpha(1-\alpha)V}}_{\text{user cost}}, \tag{51}$$

where λ_t is the multiplier on the aggregate Phillips curve, and

$$\mathcal{B}_t \equiv \Big\{ \alpha V \Big[G_t - \beta^{-1} G_{t-1} + \hat{r}_t - \mathbb{E}_t \Delta \hat{q}^I_{t+1} \Big] - \kappa \big(\lambda_t - \beta^{-1} \lambda_{t-1} \big) \Big\}, \tag{52}$$

captures the contribution of durable capital to optimal policy, so that the user-cost term vanishes if $\delta = 1$.

Proof. See Appendix D.3. □

The first term, *price stabilization*, reflects both sectoral inflation and the relative price of investment goods. The planner weighs inflation by the social value of each sector, since price dispersion wastes more resources in sectors with larger sales. Price increases also cascade through the investment network, raising investment costs for downstream sectors. Movements in the investment price also alter the rental rate relative to the wage, and thus the factor mix. This term is the only one that would appear in the optimal policy rule of a production network.

The second term, *user cost*, is a novel implication of the investment network and has no counterpart in standard production networks with only intermediate inputs. It arises because capital goods are durable: the interest rate affects the rental rate through the value of undepreciated capital, while expected capital gains affect the cost of using it. This term vanishes when capital lasts only one period.

We next show how both terms vary with the investment upstreamness of the sector hit by the shock. To obtain the joint result, we add two further restrictions:

(x) *Customer-invariant investment network*: $\omega_{I,x\to s} = \omega_{I,x}$ for every customer s, so that every sector purchases the same investment bundle. This restricts the shape of the investment network, so that social value and investment upstreamness rank in the same way. However, it still preserves heterogeneity in the investment network and the presence of investment hubs.

(xi) *Capital share and price stabilization*: we assume that the capital intensity $1-\alpha$ is sufficiently small and that condition (D.23) in Appendix D.4 holds.

Proposition 3 (Optimal policy and investment hubs)**.** *Under these restrictions, consider a positive cost-push innovation to sector s. With durable capital, $\delta < 1$, the impact responses of the two terms in Equation (51) satisfy*

$$\partial_{v_s}\left[\frac{\alpha(1-\alpha)V\,\mathbb{E}_0\Delta\hat{q}_1^I - \alpha\epsilon\,\mathbb{E}_0\sum_j v_j\pi_{j,1}}{1+\alpha(1-\alpha)V}\right] > 0, \qquad \partial_{v_s}\left[\frac{\frac{\beta(1-\delta)}{1-\beta(1-\delta)}(1-\alpha)\,\mathbb{E}_0\Delta\mathcal{B}_1}{1+\alpha(1-\alpha)V}\right] < 0. \tag{53}$$

Thus, as investment upstreamness rises, the price-stabilization term calls for more tightening and the user-cost term for more accommodation.

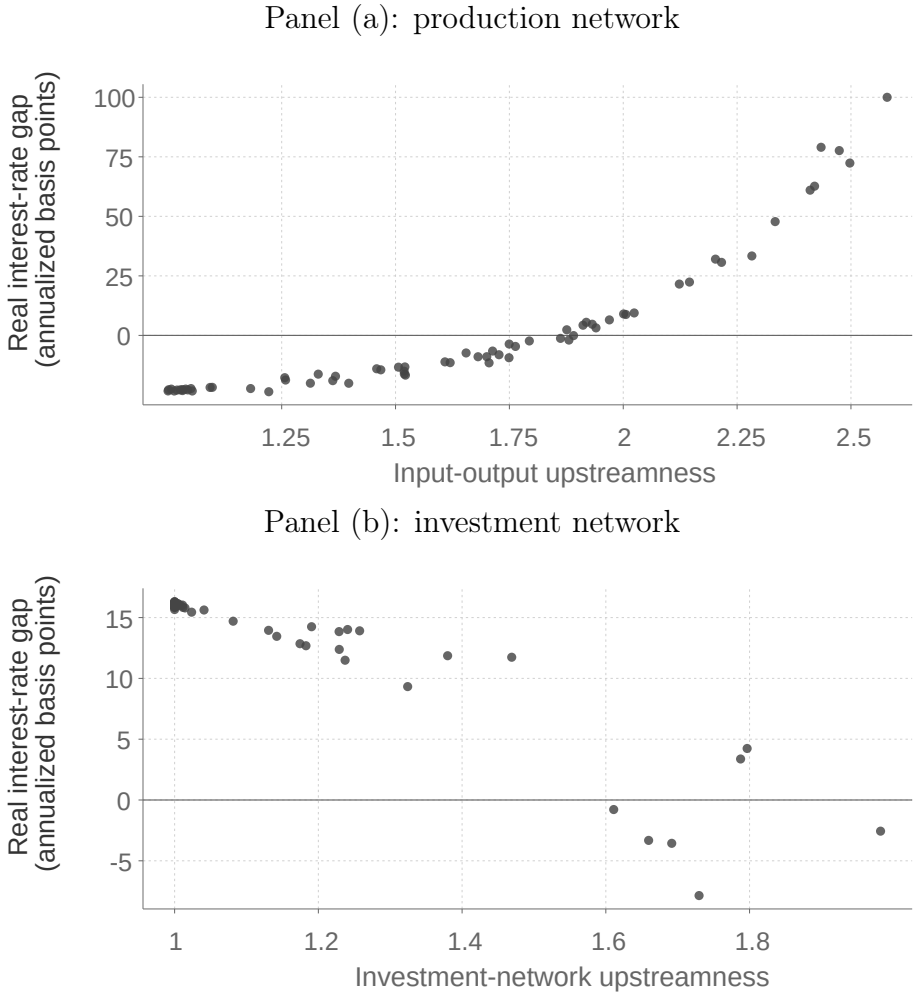
Proof. See Appendix D.4. □

Proposition 3 shows that, on impact, both channels strengthen along the investment network, but push the interest rate in opposite directions. A shock to a sector with a larger social value calls for more tightening through the price-stabilization term, as in production-network economies (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). At the same time, a shock to a larger investment hub raises the price of the common investment bundle by more, so the user-cost term calls for more accommodation. The latter contribution therefore becomes more negative as upstreamness rises. Which force dominates determines whether the planner tightens or accommodates more upon the shocks that hit investment-upstream sectors. We quantify the overall policy response in Section 6.2.

6.2 Quantitative Analysis

We now quantify how the investment network shapes optimal policy in the calibrated economy. To put its prescription into context, we compare it with that of the production network. We compute the policy under commitment after a cost-push shock to each sector in two economies. In the first economy, only the production network is heterogeneous, and everything else is homogeneous. In the second, only the investment network is heterogeneous, and everything else is homogeneous. Shock sizes are normalized so that a shock to the most input–output-upstream sector raises the annualized real interest-rate gap, that is, the deviation of the real interest rate from its flex-price value, by 100 bp in the first economy.

Figure 3: Optimal Policy across Production and Investment Networks.

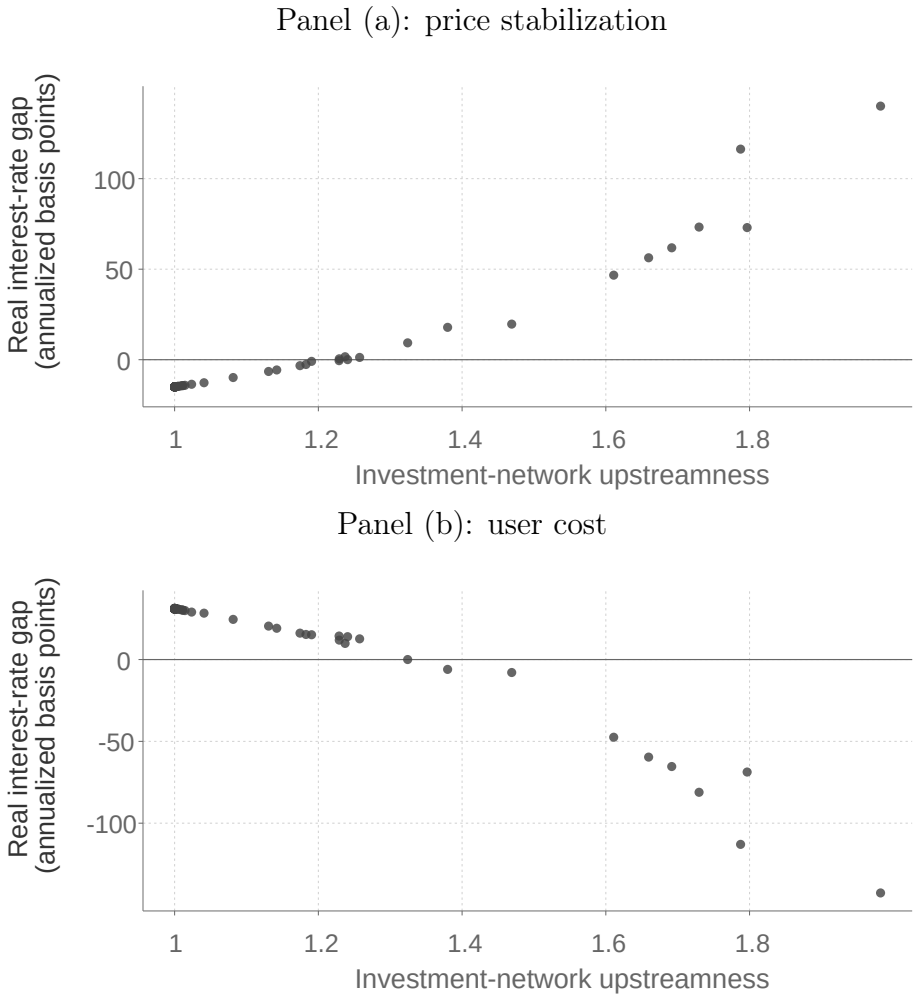


Note: Each dot represents a sector-specific inflationary shock. The vertical axis is the optimal impact real interest-rate gap in annualized basis points. The two panels differ in which network is heterogeneous; the other network and all other sectoral characteristics are homogeneous in both economies.

Figure 3 reports the results. Panel (a) shows that the planner tightens more after shocks to input–output-upstream sectors, in line with La'O and Tahbaz-Salehi (2022) and Rubbo (2023). The rate gap goes from −23 basis points for the most downstream sector to 100 bp for the most upstream one, a difference of 123 bp. Panel (b) shows the opposite pattern for the investment network. The rate gap falls from 16 bp for the most investment-downstream sector to −4 bp for the most upstream one. Hence, the investment network calls for accommodating, rather than leaning against, shocks to the sectors that supply the capital goods of the economy. The difference along this network is 20 bp, smaller than along the production network.

To disentangle the price-stabilization and user-cost effects, we consider an alternative economy in which capital fully depreciates. In this economy, the user-cost component is not active, so the optimal policy response isolates price stabilization. The difference between the baseline and full-depreciation economies then isolates the user-cost component.

Figure 4: Price Stabilization and User Cost in the Investment Network.



Note: Each dot represents a sector-specific inflationary shock. Panel (a) plots the optimal impact real interest-rate gap with full depreciation. Panel (b) plots the difference between the investment-network economy and the full-depreciation economy. Both are in annualized basis points and sum to Panel (b) of Figure 3.

Figure 4 shows this comparison. With full depreciation, the optimal rate gap rises by about 155 bp from the most investment-downstream to the most investment-upstream sector (Panel a), in line with the price-stabilization force. The user-cost term moves in the opposite direction, lowering the gap by about 175 bp across the same sectors (Panel b). Together, these responses yield the 20 bp decline in Panel (b) of Figure 3. This confirms the prediction of Proposition 3 and explains why shocks to investment-upstream sectors call for relatively more accommodation.

We finally ask whether these differences in policy matter for welfare. We implement in the fully heterogeneous economy the policy of a planner who treats either the investment or the production network as homogeneous. The extra welfare loss from ignoring the investment network is then expressed relative to the loss from ignoring the production network, separately for sectoral productivity and cost-push shocks.

To discipline the correlation across shocks, we follow La'O and Tahbaz-Salehi (2022).

Table 2: Welfare Cost of Ignoring the Investment Network.

	Productivity shocks	Cost-push shocks
Correlated shocks	75%	86%
Independent shocks	197%	149%

Note: Each cell is the welfare loss from ignoring the investment network divided by the loss from ignoring the production network, expressed as a percentage. The denominator is normalized to 100 within each shock and covariance specification.

We take logs of the annual sectoral productivity indexes in the BEA–BLS Integrated Industry-Level Production Account, linearly interpolate them to quarterly frequency, remove a sector-specific linear trend, and compute the correlation matrix of the resulting series. We also consider independent shocks: in this case, we set the cross-sector correlations to zero while keeping each sector's shock variance equal to that in the empirical case. We do not report the resulting covariance matrices separately.

Ignoring the input–output network entails a welfare loss equivalent to a permanent reduction in consumption of about 0.5 percent. What matters for our exercise, however, is the loss from ignoring the investment network relative to this benchmark. Table 2 reports these relative losses. With correlated shocks, ignoring the investment network costs 75 percent as much as ignoring the production network for productivity shocks and 86 percent for cost-push shocks. With independent shocks, these figures rise to 197 and 149 percent, respectively. Thus, abstracting from investment linkages can be at least as costly as abstracting from input–output linkages when setting monetary policy.

Overall, these results indicate not only that the investment network calls for an optimal policy response opposite to that implied by the input–output network, but also that ignoring this prescription leads to welfare losses similar to, and sometimes larger than, those from ignoring input–output linkages. The investment network therefore matters also for optimal monetary policy.

7 Concluding Remarks

This paper has studied the transmission of monetary policy through the lens of the investment network, that is, the linkages that arise from firms' sourcing of capital goods. Our results show that the real effects of monetary policy shocks are stronger in the industries that are more investment upstream, whereas the inflation responses are not statistically different across industries.

A multi-sector New Keynesian model rationalizes this evidence: a monetary tightening lowers the investment orders of every industry, and the resulting drop in demand cascades through the suppliers of capital goods, so that the output contraction is stronger the more investment-upstream a sector is. Importantly, the model accounts almost entirely for the

differential we estimate in the data, whereas an economy with a homogeneous investment network delivers no relevant differential. The investment network accounts for about 70 percent of the amplification of the aggregate GDP response to monetary policy generated by sectoral heterogeneity.

The investment network also shapes optimal monetary policy: while input–output linkages call for leaning against the inflation of upstream sectors, investment linkages call for accommodating it, because a shock to an investment hub raises the user cost of capital. In addition, the welfare cost of ignoring the capital-good flows is comparable to that of ignoring the Input-Output matrix. In other words, the investment network matters as much as the production network for the conduct of monetary policy.

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A More on the Empirical Evidence

This appendix provides further details on the data and examines the robustness of the empirical results in Section 2. We also report additional evidence on the propagation of monetary policy through the investment network.

A.1 Data

Let us start by providing additional information on the data sources. The quarterly series of real gross output, real value added, and their price indexes all come from the BEA GDP-by-Industry accounts for 66 industries, which correspond to the three-digit NAICS classification level. In setting up the list of sectors, we explicitly exclude public-sector industries. Real gross output and real value added are chain-type quantity indexes, and sectoral inflation is the annualized quarterly log change of the corresponding chain-type price index.

Investment and input–output upstreamness. To construct the investment network, we need to know which capital goods each industry purchases, how much of each good is imported, and which industries produce the domestic part. We obtain this information from the BEA Fixed Assets and Input–Output Tables for 1997. In what follows, we denote by $K^k_{c\to s}$ the purchases of commodity c by industry s in investment category k, after the industry mapping described below, ι^k_c is its imported share, and $\mu_{c\to x}$ is the share of its domestic production attributable to industry x.

We proceed in five steps. We first construct investment purchases by capital commodity and using industry, following vom Lehn and Winberry (2022). We then map these flows to the 66 industries of our panel, remove imports, allocate domestic purchases to producing industries, and divide the resulting flows by each supplier's gross output. We denote the matrix of bilateral flows by $\mathbf{F}^I = [F^I_{x\to s}]$, where $F^I_{x\to s}$ is the value of capital goods produced by industry x and purchased by industry s.

In the first step, we construct separate flow tables for equipment, non-residential structures, intellectual property products, and residential investment. We allocate investment expenditures by asset type from the BEA Fixed Assets Tables to capital commodities using the BEA equipment bridge and the asset-to-commodity mappings of vom Lehn and Winberry (2022). Residential investment comes from the corresponding private-fixed-investment column of the Use Table. We also apply their adjustments for used and scrap equipment and for margins on transactions in existing housing.

In the second step, we map capital commodities and using industries to the 66 industries of the quarterly data. We map purchasing industries in the Fixed Assets Tables to our panel and assign residential investment to owner-occupied housing. Where information, retail trade, and transportation are available only at a more aggregated level, we split these supplying categories using asset-specific shares from the Use Table and investment bridge.

In the third step, we remove imports separately for each investment category. For each commodity, we divide imports of that category of private fixed investment in the 1997 import matrix by the corresponding private-fixed-investment use in the 1997 Use Table. We remove this imported share from the purchases of every industry.

In the fourth step, we allocate domestic purchases to producing industries using $\mu_{c\to x}$ from the 1997 Make Table. This assumes that the mix of industries producing a capital commodity is the same for every industry that buys it. The denominator of each Make Table share includes production by industries outside this sample, so we do not assign their capital-goods sales to one of the 66 industries. The resulting flow of capital goods from industry x to industry s is therefore

$$F^{I}_{x\to s} = \sum_{k}\sum_{c} \mu_{c\to x}(1-\iota^{k}_{c})K^{k}_{c\to s}.$$

In the fifth step, we obtain the network shares by placing suppliers in rows and customers in columns. vom Lehn and Winberry (2022) divide each customer column by that customer's total investment expenditures to obtain purchasing shares. For our measure of investment upstreamness, we instead divide each supplier row by that supplier's 1997 gross output, computed from the Make Table, and use the resulting matrix in Equation (2).

We construct the matrix of intermediate-input flows, $\mathbf{F}^{H} = [F^{H}_{x\to s}]$, in the same orientation from the 1997 BEA Use Table. We subtract imports from each commodity–user flow, allocate the domestic flow to producing industries with the same Make Table shares, aggregate to the 66 industries, and divide by supplier gross output to obtain Ψ^{H}. The correlation between investment and input–output upstreamness across industries is -0.21.

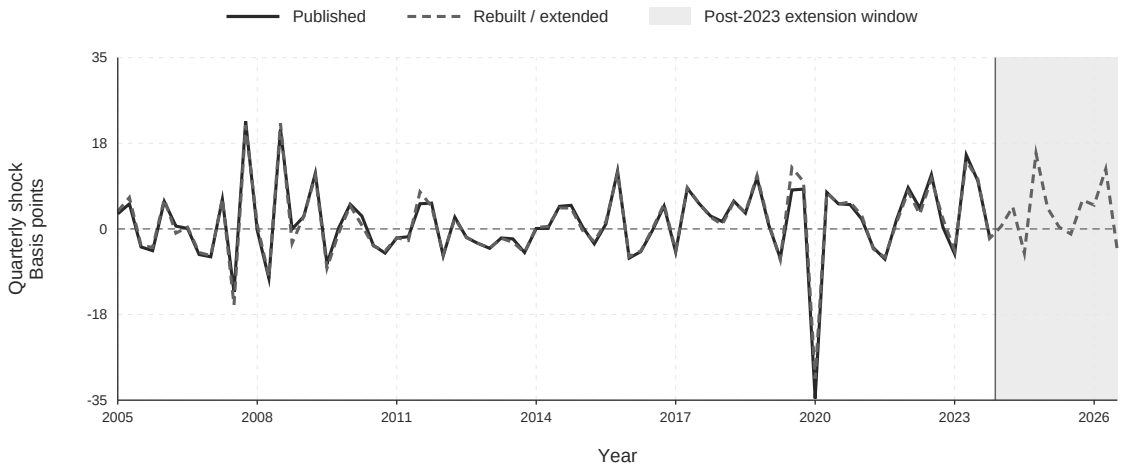
A.2 Extension of the Bauer–Swanson Shock

Our panel local projection regression in Equation (1) studies the extent to which monetary policy transmits differently across the investment network. To do so, we leverage the orthogonalized monetary policy shock of Bauer and Swanson (2023). Their series, however, ends in 2023Q4.

To extend it through 2026Q2, we closely follow the approach of Bauer and Swanson (2023). Specifically, we recover from the released workbook both the mapping from the changes in four Eurodollar futures around monetary policy announcements (ED1–ED4) to the raw surprise, and the coefficients of the monthly projection that removes its predictable component.

The six predictors are the nonfarm-payroll surprise, the twelve-month payroll growth, the three-month returns on the S&P 500 and on the Bloomberg Commodity Spot Index, the three-month change in the slope of the Treasury yield curve between ten years and three months, and the skewness of Treasury yields.

Figure A.1: Published and Reconstructed Bauer–Swanson Shocks.



Note: The figure plots the published orthogonalized monetary policy shock of Bauer and Swanson (2023) (solid line) and our reconstruction and extension (dashed line). The shaded area starts in 2024Q1. The regression sample ends in 2026Q2.

To quantify the raw surprises associated with the monetary policy announcement in the post-2023Q4 period, we apply the first set of coefficients to the announcement-window changes in ED1–ED4 from Acosta et al. (2026). We then apply the second set to updated values of the six predictors and subtract the resulting predictable component. In doing so, we keep the estimated coefficients of Bauer and Swanson (2023). Figure A.1 compares our reconstructed quarterly shock with the published one. Over the period in which both are available, their correlation is 0.99.

A.3 Value Added

We consider gross output and its price in the baseline analysis because gross output measures the entire production of an industry, including the goods it supplies as intermediate inputs to other industries. Here we show that the main finding also holds when we look at value added and its price.

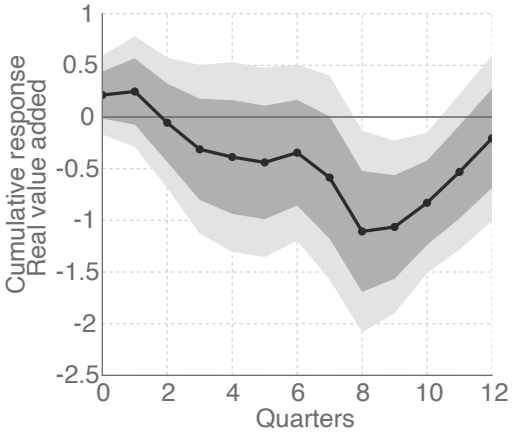
We re-estimate the panel local projections of Equation (1), taking as the dependent variable either the cumulative log change in real value added or annualized quarterly inflation of its price index. Figure A.2 reports the results.

Real value added contracts more in investment-upstream industries. The differential builds more slowly than for gross output, reaches -1.11 percentage points in the eighth quarter, and is significant at the 90-percent level from the eighth through the tenth quarter.

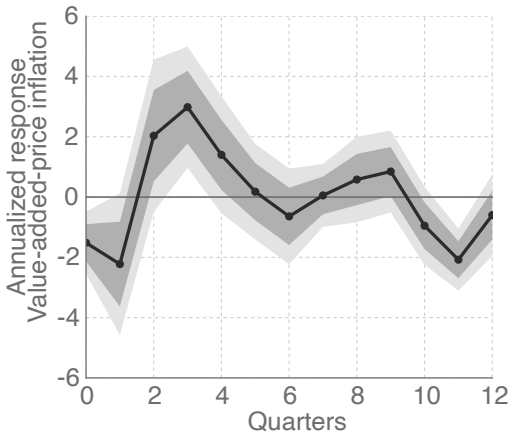
The differential in value-added-price inflation is statistically significant at some horizons, but its sign changes over time: it is negative at first, then positive, and later negative again. Overall, the estimates do not reveal a systematic relationship between investment upstreamness and the response of inflation to monetary policy shocks.

Figure A.2: Sectoral Value Added Responses.

Panel (a): Real sectoral value added



Panel (b): Sectoral value-added-price inflation



Note: The figure reports estimates of panel local projections similar to those of Figure 1, with the only difference that the dependent variables are cumulative real value added (Panel a) and annualized quarterly value-added-price inflation (Panel b).

All in all, the results for value added confirm the pattern found with gross output: investment-upstream industries contract more, while inflation has no systematic differential response. Thus, the gross-output result does not merely reflect purchases of intermediate inputs.

A.4 Alternative Explanations

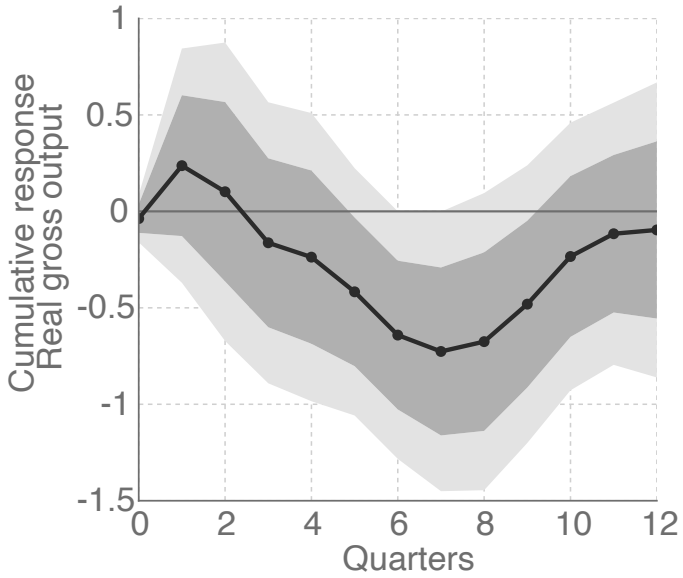
In this section, we dig deeper into the role of the investment network in the differential transmission of monetary policy. We first look at what happens when we remove the input–output interaction from the baseline regression. We then examine whether price rigidity or sectoral size can account for the estimated differential.

We start with the production network, since monetary policy also propagates through purchases of intermediate inputs (Pasten et al., 2020). When industries reduce production, they buy fewer of these inputs, affecting the sectors that supply them. The baseline regression therefore includes input–output upstreamness, so that we can compare the differential responses associated with the two networks and assess the role of investment upstreamness above and beyond the production network.

Figure A.3 reports the investment-upstreamness coefficient when we drop the interaction between the shock and input–output upstreamness. Its pattern is broadly similar, although the differential is smaller: it peaks at -0.73 rather than -1.02 percentage points and is significant at the 90-percent level only in the seventh quarter. The stronger contraction of investment-upstream industries thus does not depend on including the input–output control.

We then consider whether investment-upstream industries also differ in price rigidity

Figure A.3: Sectoral Responses without the Input–Output Control.



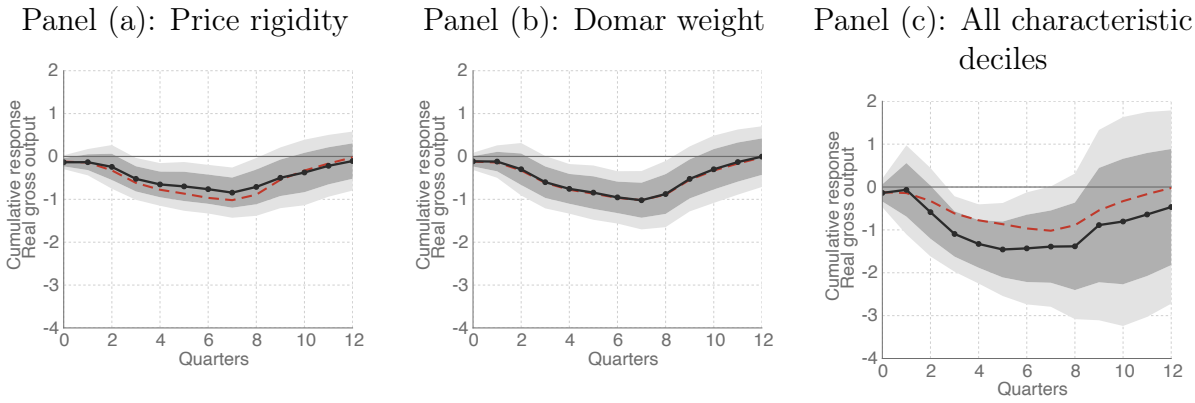
Note: The figure reports the investment-upstreamness coefficient from the panel local projections of Panel (a) of Figure 1, omitting the interaction between the shock and input–output upstreamness.

or sectoral size, as industries that supply capital goods to more customers may account for a larger share of aggregate production. The correlation between investment upstreamness and price rigidity is positive, at 0.17, while its correlation with the Domar weight is 0.10. Although these correlations are very limited, they could imply a potential threat that these alternative characteristics could act as confounding factors. To ascertain the role of the investment upstreamness in isolation, we consider two additional cases.

First, Panels (a) and (b) of Figure A.4 add a third interaction between the shock and either price rigidity, measured as one minus the monthly frequency of price changes of Pasten et al. (2020), or sectoral size, measured by the 1997 Domar weight. Like the upstreamness interactions, the additional interaction enters contemporaneously and with four lags. The investment-upstreamness differential is slightly smaller with the price-rigidity control and barely changes with the Domar-weight control. In both cases, it remains significant at the 90-percent level up to the eighth quarter.

Second, we take a more demanding approach and add to the baseline quarter fixed effects for each decile of input–output upstreamness, price rigidity, and the Domar weight. These fixed effects go beyond the baseline 2-digit NAICS-sector-by-quarter effects: they allow industries with similar positions in the production network, price rigidity, or size to respond differently to developments in each quarter. Panel (c) of Figure A.4 shows that the investment-upstreamness differential becomes larger and peaks earlier, at -1.46 percentage points in the fifth quarter, and remains significant from the third through the sixth quarter.

Figure A.4: Controlling for Other Sectoral Characteristics.



Note: The figure reports estimates of panel local projections similar to those of Panel (a) of Figure 1, with the only difference that Panels (a) and (b) add the interaction between the shock and either price rigidity or the Domar weight, and Panel (c) adds quarter fixed effects specific to each decile of input–output upstreamness, price rigidity, and the Domar weight. The red dashed line is the baseline estimate.

All in all, these results establish that investment upstreamness shapes the real transmission of monetary policy above and beyond the production network, price rigidity, and sectoral size.

A.5 Fixed Effects and Control Set

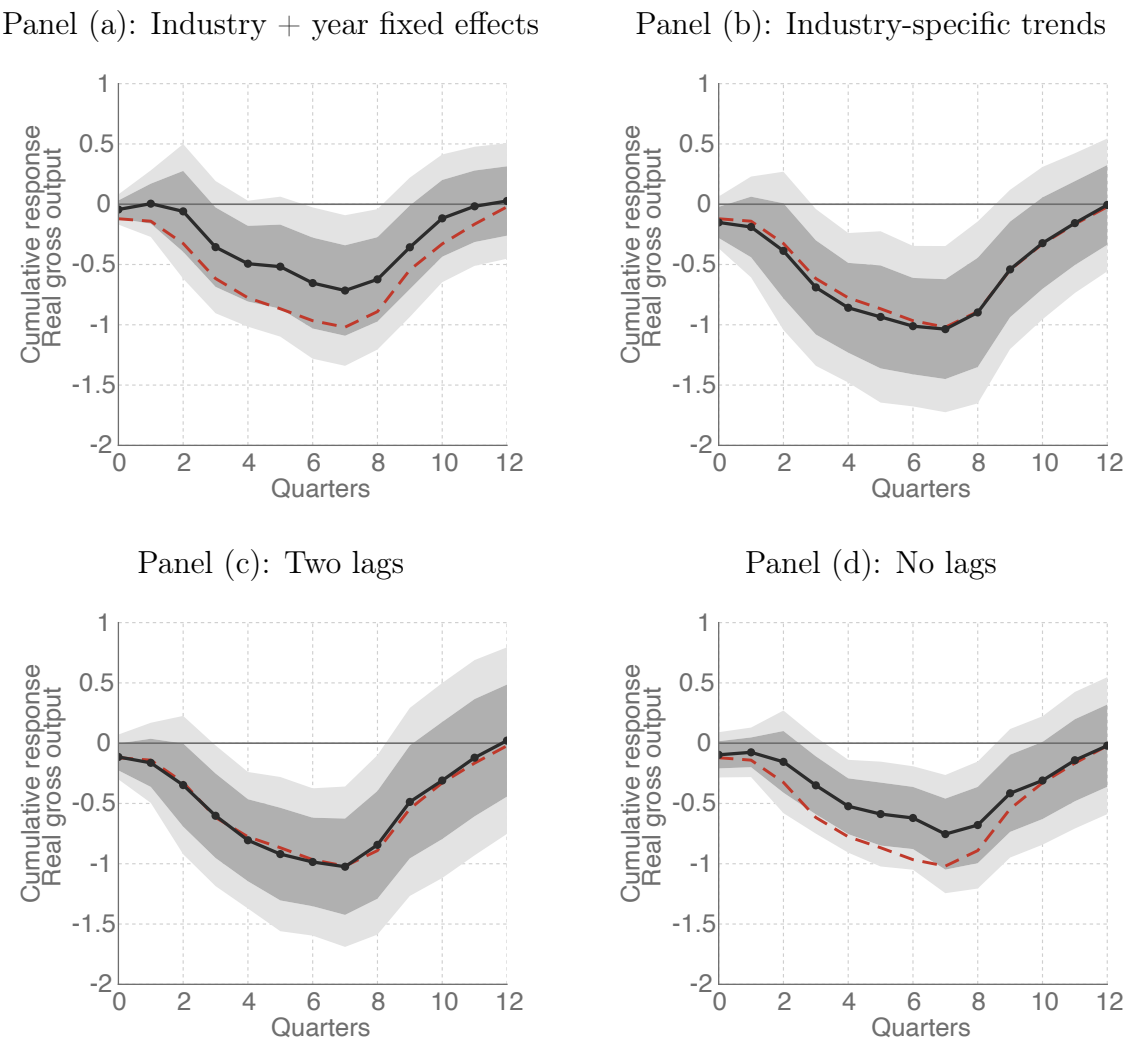
The baseline specification of Equation (1) features industry-by-quarter-of-year and 2-digit NAICS-sector-by-quarter fixed effects. The former capture industry-specific seasonal patterns, while the latter absorb developments common to industries of the same broad sector in each quarter. Here we ask whether the investment-upstreamness differential depends on these fixed effects or the number of lags in the regression.

We first change the fixed effects. Panel (a) of Figure A.5 replaces them with industry and calendar-year fixed effects. In this case, identification also exploits variation across 2-digit NAICS sectors, and the regression no longer absorbs developments common to industries of the same sector in a given quarter. The differential is smaller, with a peak of -0.72 percentage points, but remains significant from the sixth through the eighth quarter.

We then add industry-specific linear trends to the baseline fixed effects in Panel (b). These trends absorb gradual changes in each industry's activity over the sample. The estimated differential barely changes.

Finally, we change the number of lags. The baseline controls for four lags of the dependent variable and of both interactions. Panels (c) and (d) keep the same sample but include two lags and no lags, respectively. With two lags, the differential barely changes. Without lags, it is smaller, with a peak of -0.75 percentage points, but remains significant at the 90-percent level from the fourth through the eighth quarter. Thus, the stronger contraction of investment-upstream industries holds across these fixed-effect and lag specifications.

Figure A.5: Alternative Fixed Effects and Control Sets.



Note: The figure reports estimates of panel local projections similar to those of Panel (a) of Figure 1, with the only difference that Panel (a) replaces the baseline fixed effects with industry and calendar-year fixed effects, Panel (b) adds industry-specific linear trends to the baseline fixed effects, and Panels (c) and (d) include two lags and no lags of all the regressors. The red dashed line is the baseline estimate.

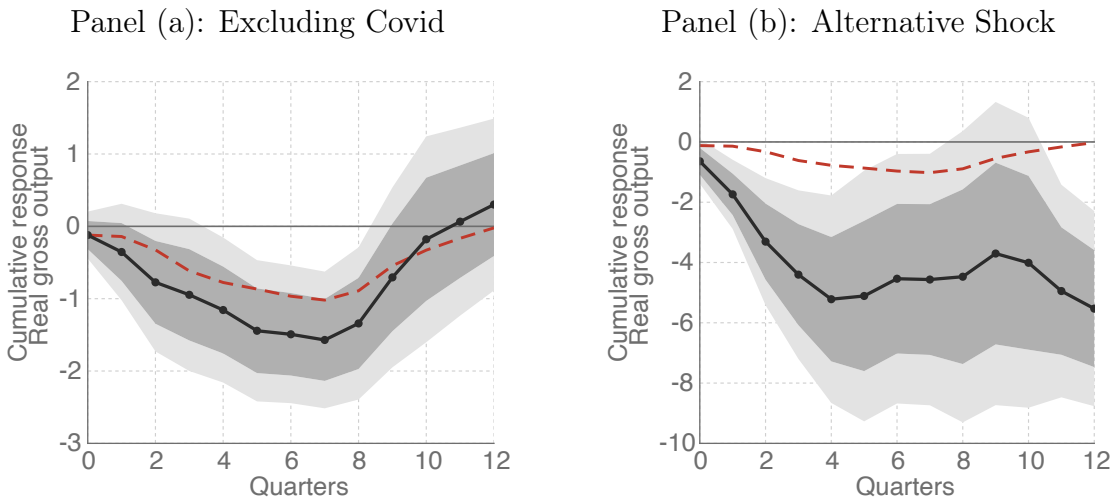
A.6 Monetary Policy Shock and Sample

A natural concern is that the investment-upstreamness differential depends on the pandemic period at the end of the sample or the monetary policy shock we use. We address these two concerns by restricting the sample and changing the shock.

We first consider the baseline shock of Bauer and Swanson (2023) and restrict the sample to responses observed by 2019Q4, so that we rule out the pandemic period. Next, we use the shock of Miranda-Agrippino and Ricco (2021), extended through 2019Q2 by Degasperi and Ricco (2021). We sum the monthly shocks within each quarter and divide the quarterly series by its standard deviation. The estimation sample ends in 2019Q2.[10]

[10]We cannot extend this shock through 2026Q2 because its construction uses the Tealbook forecasts of the Federal Reserve, which are released with a five-year lag.

Figure A.6: Alternative Sample and Monetary Policy Shock.



Note: The figure reports estimates of panel local projections similar to those of Panel (a) of Figure 1. Panel (a) uses the baseline shock of Bauer and Swanson (2023) and excludes responses observed after 2019Q4. Panel (b) uses the shock of Miranda-Agrippino and Ricco (2021), with the sample through 2019Q2, and reports the response for a one-standard-deviation increase in investment upstreamness to a 25bp tightening. In both panels, the solid line with circles is the point estimate, and the darker and lighter bands are the pointwise 68- and 90-percent Driscoll–Kraay confidence intervals. The red dashed line in Panel (a) is the baseline estimate.

Panel (a) of Figure A.6 reports the exercise excluding responses observed after 2019Q4, and Panel (b) reports the alternative-shock exercise. We find that the gross-output differential remains negative and statistically significant. If anything, it becomes more negative, and especially so for the case of the Miranda-Agrippino and Ricco (2021) shock.

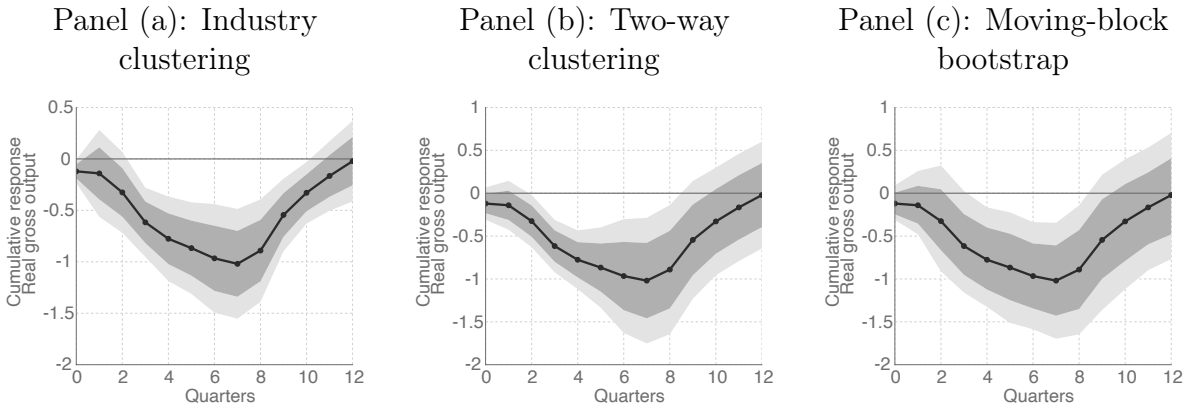
A.7 Inference

The baseline regression uses Driscoll–Kraay standard errors with a bandwidth of $\max(4, h+1)$ quarters. A natural concern is whether the significance of the investment-upstreamness differential depends on this choice. We address it by computing confidence intervals in three other ways.

We first cluster standard errors by industry, using t critical values with 65 degrees of freedom. We then cluster by industry and quarter. Finally, we use a circular moving-block bootstrap of the quarterly scores with blocks of eight quarters and 1,999 draws, which delivers the widest bands among the block lengths we consider.

Figure A.7 reports the results. The investment-upstreamness differential remains significant at the 90-percent level in the medium run: from the third through the tenth quarter with industry clustering, from the second through the eighth quarter with two-way clustering, and from the fourth through the eighth quarter with the bootstrap. Thus, the finding does not depend on how we compute standard errors.

Figure A.7: Alternative Clustering Procedures.



Note: The figure reports the baseline estimates of Panel (a) of Figure 1, with the only difference that the darker and lighter bands are the pointwise 68- and 90-percent confidence intervals with standard errors clustered by industry (Panel a), two-way clustered by industry and quarter (Panel b), and from a circular moving-block bootstrap with blocks of eight quarters (Panel c).

A.8 Network Propagation

The results above show that investment-upstream industries contract more after a monetary tightening. Yet they do not by themselves show that the response propagates through the investment network, from the industries that demand capital goods to their suppliers, the investment hubs. We provide evidence on this propagation in two ways.

First, we follow Ozdagli and Weber (2026) and estimate a spatial autoregression. It relates the output response of each supplier to the responses of its investment customers, giving more weight to customers to which the supplier sells a larger share of its output. In this way, the model can account for how the way in which the propagation via the sales and purchases of capital-goods across industries shapes sectors' responses to monetary policy shocks.

Second, we follow Acemoglu et al. (2016) and separate an industry's direct sensitivity to monetary policy from its indirect exposure to its customers' investment demand. This second approach also allows us to retain the 2-digit NAICS-sector-by-quarter fixed effects of the baseline regression.

A.8.1 Spatial Autoregression

Ozdagli and Weber (2026) use a model with intermediate inputs to motivate a spatial autoregression for the response of industry stock returns to monetary policy announcements. They use it to separate the direct response to the shock from the response that reaches an industry through its customers in the production network. We follow the same approach for real gross output, replacing intermediate-input flows with sales of capital goods.

For each horizon $h = 0, \ldots, 12$, we estimate across the 66 industries and the quarters

$$Y^h_{s,t} = \rho^h_I \sum_{x=1}^{S} \Psi^I_{sx} Y^h_{x,t} + \beta_h m_t + \mathbf{X}'_{s,t-1}\theta_h + \mu^h_{s,q(t)} + u^h_{s,t}, \tag{A.1}$$

where $Y^h_{s,t}$ is the cumulative log change in the real gross output of industry s between $t-1$ and $t+h$, and Ψ^I_{sx} is the share of the gross output of supplier s sold as investment goods to customer x, the same matrix behind investment upstreamness in Equation (2).

The shock enters directly through $\beta_h m_t$. The spatial lag $\sum_x \Psi^I_{sx} Y^h_{x,t}$ relates the response of supplier s to those of its investment customers, giving more weight to customers to which the supplier sells a larger share of its output. The spatial autoregression is useful here because these bilateral sales identify the customers through which the shock can reach each supplier. The supplier's response may then reach the industries from which it purchases capital goods, allowing the shock to move further upstream.

The controls $\mathbf{X}_{s,t-1}$ include four lags of the shock, of the output growth of the industry, and of the average output growth across industries. The regression imposes a common direct response β_h across industries and includes industry-by-quarter-of-year fixed effects $\mu^h_{s,q(t)}$.

At each horizon, Equation (A.1) estimates the direct response β_h, which holds customer outcomes fixed, and the propagation coefficient ρ^h_I. Using these estimates and the investment-network matrix, we obtain the total response of each industry as $\beta_h[(\mathsf{I} - \rho^h_I \Psi^I)^{-1}\mathbf{1}]_s$. The matrix inverse includes all rounds of capital-goods sales. We then average the total response across the 66 industries, $\beta_h\, \mathbf{1}'(\mathsf{I} - \rho^h_I \Psi^I)^{-1}\mathbf{1}/S$, and subtract the direct response β_h. This difference is the response that propagates through the investment network.

We estimate the model by concentrated maximum likelihood, horizon by horizon, and compute the confidence intervals with a moving-block bootstrap over quarters, with blocks of $h+8$ quarters and 500 draws, re-estimating the model in each draw.

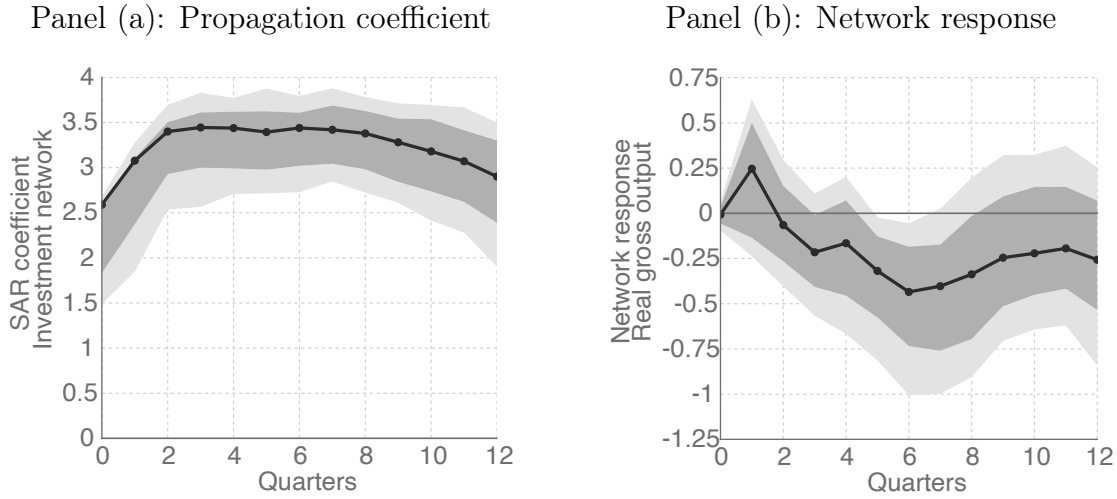
Panel (a) of Figure A.8 shows that the propagation coefficient ρ^h_I is large and precisely estimated at every horizon, between 2.6 and 3.4. This coefficient does not by itself give the share of the response due to the network, which also depends on the bilateral sales in Ψ^I.

Panel (b) reports the implied network response. At the peak of the average gross-output response, in the sixth quarter, the total response is -1.50 percent and the direct response is -1.06 percent. Their difference, -0.43 percentage points, is the network response: it accounts for about 30 percent of the total response (0.43/1.50). The network response is negative from the second quarter onward and significant at the 90-percent level in the fifth and sixth quarters. These estimates provide direct evidence of the propagation of monetary policy shocks through the investment network, from customer industries onto investment hubs.

A.8.2 Regression à la Acemoglu, Akcigit, and Kerr (2016)

The spatial autoregression traces how the response of monetary policy reaches capital-goods suppliers through their investment customers. However, the joint response of

Figure A.8: Spatial Autoregression.



Note: Panel (a) plots the estimated propagation coefficient ρ_I^h of Equation (A.1) at each horizon. Panel (b) plots the network response of real gross output averaged across industries, that is, the total response minus the direct response. The darker and lighter bands are the pointwise 68- and 90-percent moving-block bootstrap confidence intervals.

suppliers and customers could be driven by the presence of changes in common aggregate conditions. Yet, the spatial autoregression cannot rule this possibility out by including 2-digit NAICS-sector-by-quarter fixed effects, since these would also absorb the aggregate monetary policy shock.

To address this concern, we complement the spatial autoregression with the approach of Acemoglu et al. (2016), which separates an industry's direct response to a shock from the response that reaches it through the network. Their approach starts from sectoral shocks. Here the monetary policy shock is aggregate, so we construct industry-specific measures of direct and indirect exposure and interact each of them with the same shock. For each horizon $h = 0, \ldots, 12$, we estimate

$$Y_{s,t}^h = \beta_h^d \, d_s m_t + \beta_h^I \, d_s^I m_t + \mathbf{X}'_{s,t-1}\theta_h + \mu_{s,q(t)}^h + \lambda_{b(s),t}^h + \kappa_s t + \varepsilon_{s,t}^h, \tag{A.2}$$

where $Y_{s,t}^h$ is the cumulative log change in real gross output. The regression includes four lags of the dependent variable and of both interactions, the fixed effects of the baseline regression, and industry-specific linear trends, $\kappa_s t$.

The monetary policy shock is common to all industries, but their direct sensitivity to it may differ. To measure this sector-specific sensitivity, we use the calibrated model and shut down purchases across industries. Specifically, we make both networks roundabout, so that each industry buys investment and intermediate inputs only from itself:

$$\boldsymbol{\omega}_I^{\text{rb}} = \boldsymbol{\omega}_H^{\text{rb}} = \mathsf{I}. \tag{A.3}$$

We keep price rigidity, factor intensities, depreciation, and consumption shares as in the baseline calibration. Industries can therefore still respond differently to monetary policy, and purchases from their own sector still matter. What we shut down are the

investment and intermediate-input links between industries. The gross-output response in this economy thus measures each industry's sensitivity to the common shock before the response passes through these links.

We then simulate a monetary tightening in this economy. We take the average response of real gross output from impact through month 38, which corresponds to the horizons of the local projections, and reverse its sign. Hence, a higher d_s means that industry s contracts more in the roundabout economy.

To measure indirect exposure, we take the vector $\mathbf{d}$ of these direct responses and propagate it through the empirical investment network. Since d_s already includes the effect of an industry's purchases from itself, we set the diagonal of the seller-by-customer matrix Ψ^I to zero before computing

$$\mathbf{d}^I = \left[(\mathsf{I} - \Psi^{I\circ})^{-1} - \mathsf{I}\right]\mathbf{d}, \qquad \Psi^{I\circ} = \Psi^I - \operatorname{diag}(\Psi^I). \tag{A.4}$$

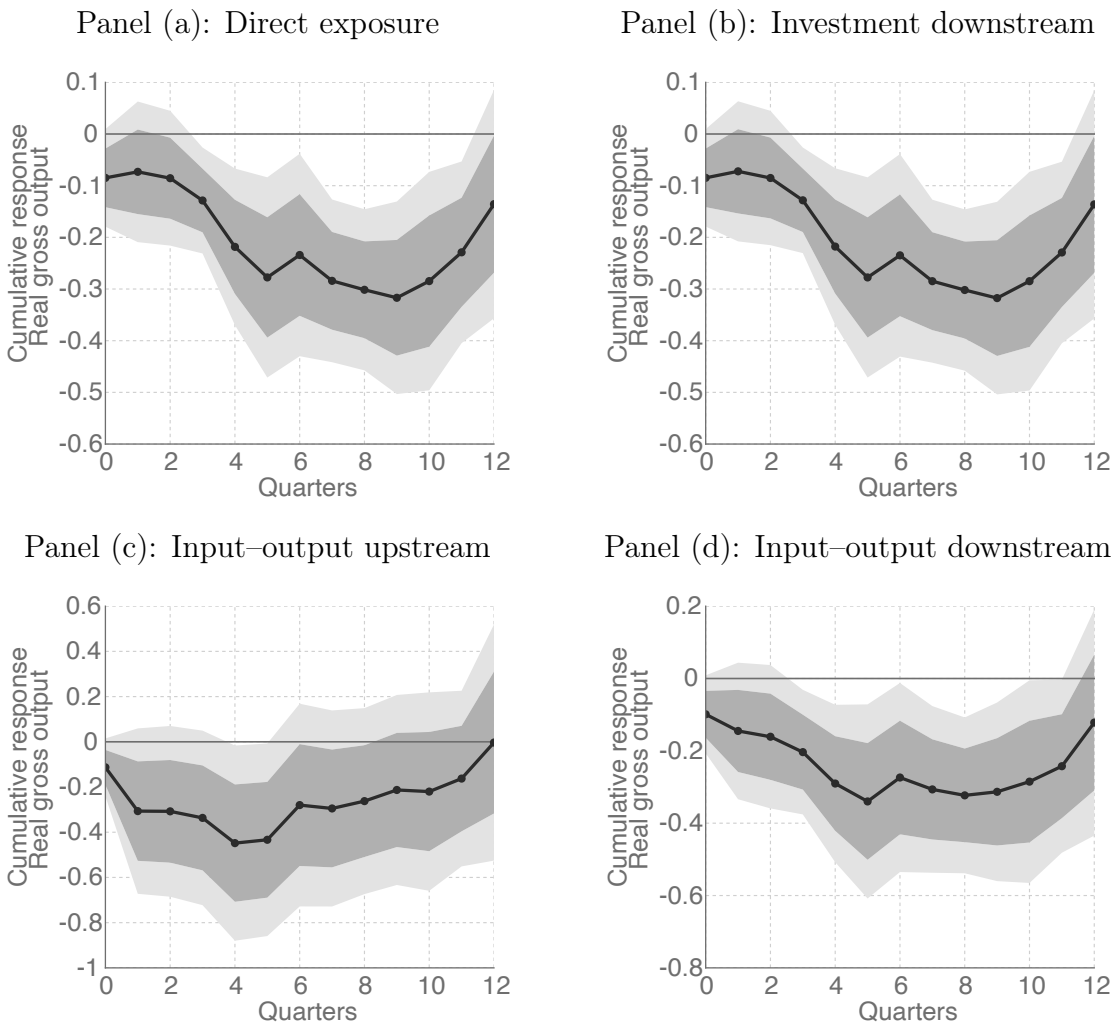
The resulting d_s^I combines the direct exposures of the investment customers of supplier s, also through successive rounds of sales. We construct input–output-upstream exposure in the same way, using the input–output network. We also construct investment- and input–output-downstream exposures from the purchases of each industry from its suppliers, divided by its gross output. We standardize all exposures across the 66 industries.

In Equation (A.2), the first interaction measures how the response varies with an industry's direct sensitivity to monetary policy. The second measures how it varies with the exposure of its investment customers, conditional on that direct sensitivity. Since both exposures differ across industries, we can retain the 2-digit NAICS-sector-by-quarter fixed effects and identify the investment-upstream response from differences within broad sectors. We then add, one at a time, the interaction of the shock with investment-downstream, input–output-upstream, or input–output-downstream exposure.

Panel (a) of Figure A.9 shows that, conditional on direct exposure, industries whose investment customers are more exposed contract more after a monetary tightening. The investment-upstreamness differential peaks at -0.32 percentage points in the ninth quarter and is significant at the 90-percent level from the third through the eleventh quarter.

The other panels add one network exposure at a time. Controlling for investment-downstream exposure leaves the investment-upstreamness differential almost unchanged. With input–output-upstream exposure, it is larger at the one-year horizon, at -0.45 percentage points in the fourth quarter, but significant only in the fourth and fifth quarters. It remains significant from the third through the eleventh quarter when we control for input–output-downstream exposure. Overall, these results corroborate the role of the investment network in monetary transmission.

Figure A.9: Sectoral Responses by Investment Upstreamness - Network Exposure Controls.



Note: All panels report the investment-upstreamness differential in real gross output from Equation (A.2). Panel (a) controls for direct exposure. Panels (b)–(d) also control for investment-downstream, input–output-upstream, and input–output-downstream exposure, respectively.

B The Model: Details and Derivations

This appendix provides additional details on the model in Section 3. We report the remaining optimality conditions, the demand schedules of wholesalers and retailers, and the market-clearing conditions.

B.1 Household

The household chooses consumption, labor, bond holdings and, for every sector, investment and the capital stock it accumulates, so as to maximize Equation (3) subject to the budget constraint (4) and the S sector-specific physical capital accumulation constraints (5). The perfect mobility of labor across sectors implies that the household supplies aggregate labor N_t at a common nominal wage W_t. Capital is instead sector specific, implying one law of motion, one investment first-order condition, and one shadow value of capital for each sector.

Let $\mathcal{M}_{t,\tau} \equiv \beta^{\tau-t}(C_t/C_\tau)^\sigma(P_t^C/P_\tau^C)$ denote the household's nominal stochastic discount factor between t and $\tau \geq t$. The first-order condition for nominal bonds is

$$1 = R_t \mathbb{E}_t \mathcal{M}_{t,t+1}. \tag{B.1}$$

The first-order condition for labor is

$$\theta N_t^\eta = \frac{W_t}{P_t^C C_t^\sigma}. \tag{B.2}$$

The first-order conditions for sectoral investment and capital are reported in Equations (6) and (7) in the main text. The former takes into account the investment adjustment costs, while the latter relates the shadow value of capital to next period's rental rate and the value of the capital that does not depreciate.

B.2 Producers

Producer i in sector s hires labor at the economy-wide wage W_t, rents sector-s capital from the household at $R_{s,t}^K$, buys its industry's intermediate bundle at $P_{s,t}^H$, and sets its variety price subject to the Calvo friction. It solves Equation (9) subject to the technology (8) and the variety demand schedule (B.9). We denote by $N_{s,t} \equiv \int_0^1 N_{s,t}^i \mathrm{d}i$ the labor employed in sector s, and define $K_{s,t}$ and $H_{s,t}$ analogously. Since all the producers of a sector face the same input prices and have the same technology, they also share the same nominal marginal cost, $\Xi_{s,t}$. Cost minimization over the three inputs implies the following first-order conditions:

$$W_t N_{s,t} = \alpha_{N,s} \Xi_{s,t} \Upsilon_{s,t} Z_{s,t}, \tag{B.3}$$

$$R_{s,t}^K K_{s,t} = \alpha_{K,s} \Xi_{s,t} \Upsilon_{s,t} Z_{s,t}, \tag{B.4}$$

$$P_{s,t}^H H_{s,t} = \alpha_{H,s} \Xi_{s,t} \Upsilon_{s,t} Z_{s,t}, \tag{B.5}$$

where $\Upsilon_{s,t} Z_{s,t} = \int_0^1 Z_{s,t}^i \mathrm{d}i$ is the total production of sector s, and $\Upsilon_{s,t} \geq 1$ denotes the price dispersion defined in Equation (B.10).

Producers set their price according to a Calvo-type pricing protocol. The probability that the price remains fixed from one period to the next, ϑ_s, is identical across the producers of the same sector. Hence, all the producers that reset their price at time t choose the same optimal reset price, $P^{\star}_{s,t}$, which satisfies the following first-order condition:

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \vartheta_s^{\tau-t} \mathcal{M}_{t,\tau} \left(\frac{P^{\star}_{s,t}}{P_{s,\tau}} \right)^{-\epsilon} Z_{s,\tau} \left[P^{\star}_{s,t} - \frac{\epsilon}{\epsilon - 1} \Xi_{s,\tau} \right] = 0, \tag{B.6}$$

where $(P^{\star}_{s,t}/P_{s,\tau})^{-\epsilon} Z_{s,\tau}$ is the demand at time τ for a variety whose price has been reset at time t. In other words, the optimal reset price is a markup over a weighted average of the current and expected future marginal costs. Without nominal rigidities, that is, $\vartheta_s = 0$, the condition collapses to a constant markup over the marginal cost. By the law of large numbers, a fraction $1 - \vartheta_s$ of producers reset their price in each period, so that the sectoral price index of Equation (B.8) evolves as

$$P_{s,t}^{1-\epsilon} = (1 - \vartheta_s) \big(P^{\star}_{s,t} \big)^{1-\epsilon} + \vartheta_s P_{s,t-1}^{1-\epsilon}. \tag{B.7}$$

Under the sales subsidy of restriction (ix), revenue is $(1+\varsigma) P^i_{s,t} Z^i_{s,t}$ with $1 + \varsigma = \epsilon/(\epsilon - 1)$, so that the markup $\epsilon/(\epsilon - 1)$ in Equation (B.6) becomes $\epsilon/[(\epsilon - 1)(1 + \varsigma)] = 1$, and the zero-inflation steady state has $\bar{\Xi}_s = \bar{P}_s$.

B.3 Wholesalers

The sectoral wholesaler buys the varieties produced in its industry and bundles them into the sectoral good with the technology in Equation (10), taking variety prices as given and earning zero profits. Its unit cost is the sectoral price index

$$P_{s,t} = \left[\int_0^1 (P^i_{s,t})^{1-\epsilon} \mathrm{d}i \right]^{1/(1-\epsilon)}, \tag{B.8}$$

where $P^i_{s,t}$ is the price charged by producer i and $\epsilon > 1$ governs substitution across varieties of the same industry.

Cost minimization gives the demand for producer i's variety,

$$Z^i_{s,t} = \left(\frac{P^i_{s,t}}{P_{s,t}} \right)^{-\epsilon} Z_{s,t}, \tag{B.9}$$

so that a producer that raises its price relative to its industry's index loses demand with elasticity ϵ.

Integrating the demand schedule across producers gives the total production of sector s, $\int_0^1 Z^i_{s,t} \mathrm{d}i = \Upsilon_{s,t} Z_{s,t}$, where $\Upsilon_{s,t} \equiv \int_0^1 (P^i_{s,t}/P_{s,t})^{-\epsilon} \mathrm{d}i$ denotes price dispersion, which evolves as

$$\Upsilon_{s,t} = (1 - \vartheta_s) \left(\frac{P^{\star}_{s,t}}{P_{s,t}} \right)^{-\epsilon} + \vartheta_s \Pi^{\epsilon}_{s,t} \Upsilon_{s,t-1}. \tag{B.10}$$

Since all the producers of a sector combine inputs in the same proportions, the technology in Equation (8) aggregates to

$$\Upsilon_{s,t} Z_{s,t} = N_{s,t}^{\alpha_{N,s}} K_{s,t}^{\alpha_{K,s}} H_{s,t}^{\alpha_{H,s}}. \tag{B.11}$$

Hence, price dispersion raises the amount of inputs required to produce a given amount of the sectoral good.

B.4 Retailers

Each retailer is perfectly competitive, takes sectoral prices as given, buys the sectoral goods that enter its aggregator, and sells the resulting bundle at unit cost. We report the cost index and the component demand implied by each of the three technologies in the main text.

Consumption retailer. The retailer operates the aggregator in Equation (12). Its unit cost is the consumption price index

$$P_t^C = \left[\sum_{x=1}^{S} \omega_{C,x} P_{x,t}^{1-\nu_C}\right]^{1/(1-\nu_C)}, \tag{B.12}$$

where $P_{x,t}$ is the price of the good supplied by sector x and ν_C is the elasticity of substitution across those goods. Cost minimization implies the expenditure shares

$$\frac{P_{x,t} C_{x,t}}{P_t^C C_t} = \frac{\omega_{C,x} P_{x,t}^{1-\nu_C}}{\sum_{y=1}^{S} \omega_{C,y} P_{y,t}^{1-\nu_C}}, \tag{B.13}$$

and the demand for the sectoral consumption good

$$C_{x,t} = \omega_{C,x} \left(\frac{P_{x,t}}{P_t^C}\right)^{-\nu_C} C_t. \tag{B.14}$$

Investment retailers. For each customer s, the retailer operates the aggregator in Equation (13) and sells at its unit cost, the CES price index

$$P_{s,t}^I = \left[\sum_{x=1}^{S} \omega_{I,x\to s} P_{x,t}^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{B.15}$$

Cost minimization implies the expenditure shares

$$\frac{P_{x,t} I_{x\to s,t}}{P_{s,t}^I I_{s,t}} = \frac{\omega_{I,x\to s} P_{x,t}^{1-\nu_I}}{\sum_{y=1}^{S} \omega_{I,y\to s} P_{y,t}^{1-\nu_I}}, \tag{B.16}$$

and the demand for supplier x's contribution to customer s's investment bundle

$$I_{x\to s,t} = \omega_{I,x\to s} \left(\frac{P_{x,t}}{P_{s,t}^I}\right)^{-\nu_I} I_{s,t}. \tag{B.17}$$

The household purchases $I_{s,t}$ units of this bundle. The investment adjustment costs affect capital accumulation, but not the retailer's demand for sectoral goods.

Intermediate-input retailers. For each customer s, the retailer operates the aggregator in Equation (14). Its unit cost is

$$P_{s,t}^H = \left[\sum_{x=1}^{S} \omega_{H,x\to s} P_{x,t}^{1-\nu_H}\right]^{1/(1-\nu_H)}, \tag{B.18}$$

where ν_H is the elasticity of substitution across its suppliers. Cost minimization implies

the expenditure shares

$$\frac{P_{x,t}H_{x\to s,t}}{P^H_{s,t}H_{s,t}} = \frac{\omega_{H,x\to s}P^{1-\nu_H}_{x,t}}{\sum_{y=1}^{S}\omega_{H,y\to s}P^{1-\nu_H}_{y,t}}, \tag{B.19}$$

and the component demand

$$H_{x\to s,t} = \omega_{H,x\to s}\left(\frac{P_{x,t}}{P^H_{s,t}}\right)^{-\nu_H} H_{s,t}. \tag{B.20}$$

The matrix $\boldsymbol{\omega}_H = [\omega_{H,x\to s}]$ has supplier industries in its rows and customer industries in its columns, just like the investment network $\boldsymbol{\omega}_I$.

The analytical results also use the share of supplier x's steady-state gross output sold as investment goods to customer s:

$$\psi_{x\to s} \equiv \frac{\bar{I}_{x\to s}}{\bar{Z}_x}. \tag{B.21}$$

The matrix $\Psi^I = [\psi_{x\to s}]$ has suppliers in its rows and customers in its columns. Both matrices use the same bilateral investment flows, but Ψ^I divides them by each supplier's gross output, whereas the weights $\boldsymbol{\omega}_I$ are chosen to reproduce the expenditure shares of each customer. Hence, Ψ^I is not the transpose of $\boldsymbol{\omega}_I$.

B.5 Market Clearing and Aggregation

Labor is perfectly mobile, so a single market clears it. Total hours supplied by the household equal total hours hired across producers and industries,

$$N_t = \sum_{s=1}^{S}\int_0^1 N^i_{s,t}\mathrm{d}i. \tag{B.22}$$

Capital is sector specific, so the household's supply of sector-s capital equals the quantity rented by producers in that sector,

$$K_{s,t} = \int_0^1 K^i_{s,t}\mathrm{d}i. \tag{B.23}$$

The one-period nominal bond is in zero net supply,

$$B_t = 0. \tag{B.24}$$

The sectoral goods markets clear according to Equation (11). Component purchases are $I_{x\to s,t}$, and the investment retailer of customer s aggregates them into $I_{s,t}$. The investment adjustment costs affect the capital accumulated from that bundle without creating a separate demand for goods. Wholesalers and retailers earn zero profits, so aggregate nominal dividends are the profits of producers,

$$\mathcal{D}_t = \sum_{s=1}^{S}\Big[P_{s,t}Z_{s,t} - W_tN_{s,t} - R^K_{s,t}K_{s,t} - P^H_{s,t}H_{s,t}\Big]. \tag{B.25}$$

Sectoral real gross output is $Z_{s,t}$, and nominal gross output is $P_{s,t}Z_{s,t}$. Sectoral nominal value added subtracts intermediate-input purchases from nominal gross output, as in

Equation (16). We define sectoral real value added, $Y_{s,t}$, by valuing gross output and intermediate inputs at their steady-state prices:

$$Y_{s,t} = \bar{P}_s Z_{s,t} - \bar{P}_s^H H_{s,t}. \tag{B.26}$$

Summing sectoral nominal value added and using goods-market clearing gives nominal GDP,

$$\mathcal{Y}_t = \sum_{s=1}^{S} \mathcal{Y}_{s,t} = P_t^C C_t + \sum_{s=1}^{S} P_{s,t}^I I_{s,t},$$

as in Equation (17). Intermediate-input sales cancel with the corresponding purchases across sectors. Real GDP values consumption and investment at steady-state prices, with the consumption price as numeraire:

$$Y_t = C_t + \sum_{s=1}^{S} \frac{\bar{P}_s^I}{\bar{P}^C} I_{s,t},$$

as in Equation (18).

C Monetary Transmission: Derivations and Proofs

This appendix reports the derivations behind the analytical results on monetary transmission in Section 5.1. We first log-linearize the equilibrium conditions and show when relative prices remain at their steady state after a monetary policy shock. We then prove Proposition 1 and Corollary 1, and derive the steady-state network accounting identities used in the analysis.

C.1 Log-Linear Conditions

This subsection log-linearizes the optimality conditions of Appendix B around the deterministic zero-inflation steady state, and reports the conditions used in the analytical results. A hat denotes the log deviation of a variable from that steady state, so that $\hat{C}_t \equiv \log(C_t/\bar{C})$ and, likewise, $\hat{Z}_{s,t}$, $\hat{K}_{s,t}$, $\hat{I}_{s,t}$, $\hat{N}_{s,t}$ and $\hat{N}_t$ are the log deviations of $Z_{s,t}$, $K_{s,t}$, $I_{s,t}$, $N_{s,t}$ and N_t; $\hat{C}_{x,t}$ and $\hat{I}_{x\to s,t}$ are those of the component demands, $\pi_{s,t} \equiv \log \Pi_{s,t}$ is sectoral inflation, and $\hat{\Xi}_{s,t}$ is the log deviation of $\Xi_{s,t}/P_{s,t}$. Relative prices carry the consumption price as numeraire, so that $\hat{q}_{x,t}$ is the log deviation of $q_{x,t} = P_{x,t}/P_t^C$, $\hat{q}^I_{s,t}$ that of $q^I_{s,t} = P^I_{s,t}/P_t^C$, and $\hat{q}^K_{s,t}$ that of $q^K_{s,t} = Q^K_{s,t}/P_t^C$. The scalar $\hat{r}_t \equiv \log R_t - \mathbb{E}_t\pi_{t+1} - \log \bar{R}$, with $\pi_t \equiv \boldsymbol{\omega}_C'\boldsymbol{\pi}_t$ consumption-price inflation, is the ex-ante real-interest-rate deviation, $\hat{w}_t$ is the log deviation of the real wage W_t/P_t^C, and $\hat{r}^K_{s,t}$ the log deviation of $R^K_{s,t}/P_t^C$.

Equation (B.1) gives the consumption Euler equation

$$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} - \frac{1}{\sigma}\hat{r}_t, \tag{C.1}$$

and Equation (B.2) gives the labor-supply condition

$$\hat{w}_t = \sigma\hat{C}_t + \eta\hat{N}_t, \tag{C.2}$$

where $\hat{w}_t$ is the log deviation of the real wage W_t/P_t^C. Because labor is perfectly mobile, one wage and one quantity enter this condition. Under the preference restrictions of Section 5.1.1 it reduces to Equation (27).

Under restriction (vii), the unit installed at t is already productive at t, and the first-order capital law is

$$\hat{K}_{s,t} = (1-\delta_s)\hat{K}_{s,t-1} + \delta_s\hat{I}_{s,t}, \qquad \hat{K}_{s,-1} = 0. \tag{C.3}$$

When $\delta_s < 1$, part of last period's capital stock remains in use today.

Linearizing Equations (6) and (7) at a steady state with constant investment, where the adjustment cost and its first derivative vanish, gives the Tobin's-q system. The investment condition implies

$$\hat{q}^K_{s,t} - \hat{q}^I_{s,t} = \phi\left[(\hat{I}_{s,t} - \hat{I}_{s,t-1}) - \beta\mathbb{E}_t(\hat{I}_{s,t+1} - \hat{I}_{s,t})\right], \tag{C.4}$$

and, writing $b_s \equiv \beta(1-\delta_s)$ for the steady-state weight on the continuation value of capital

and $1 - b_s$ for the weight on its rental return, the asset-value recursion implies

$$(1 - b_s)\mathbb{E}_t \hat{r}^K_{s,t+1} = \hat{r}_t + \hat{q}^K_{s,t} - b_s \mathbb{E}_t \hat{q}^K_{s,t+1}. \tag{C.5}$$

Under restriction (vii), the first-order condition for $K_{s,t}$ is $Q^K_{s,t} = R^K_{s,t} + (1-\delta_s)\mathbb{E}_t[\mathcal{M}_{t,t+1} Q^K_{s,t+1}]$, in which the current rental is collected before discounting, so that $\bar{Q}^K_s(1 - b_s) = \bar{R}^K_s$ and its linearization, deflated by the consumption price, is

$$\hat{q}^K_{s,t} = (1 - b_s)\hat{r}^K_{s,t} + b_s \mathbb{E}_t \hat{q}^K_{s,t+1} - b_s \hat{r}_t. \tag{C.6}$$

The real rate now discounts the continuation value alone, because the current rental is collected before any discounting takes place. Restriction (vii) also removes the adjustment cost, so Equation (C.4) gives $\hat{q}^K_{s,t} = \hat{q}^I_{s,t}$. Substituting that equality and multiplying by $D_s \equiv (1 - b_s)^{-1}$ delivers the user cost

$$\hat{r}^K_{s,t} = D_s\left[\hat{q}^I_{s,t} - b_s \mathbb{E}_t \hat{q}^I_{s,t+1}\right] + \zeta_s \hat{r}_t, \qquad \zeta_s \equiv D_s b_s, \tag{C.7}$$

When depreciation is common, $D_s = \bar{D}$ and $\zeta_s = \zeta$. If relative prices remain at their steady-state values, so do investment prices, and Equation (C.7) reduces to Equation (31). Appendix C.2 shows when relative prices remain constant. Since capital installed at t is productive at t, the user-cost condition also holds on impact.

Linearizing the reset-price condition (B.6) and the law of motion of the sectoral price index (B.7) around zero inflation gives the Phillips curve $\pi_{s,t} = \beta \mathbb{E}_t \pi_{s,t+1} + \kappa_s \hat{\Xi}_{s,t}$ with $\kappa_s = (1 - \vartheta_s)(1 - \beta\vartheta_s)/\vartheta_s$, and restriction (iv) makes the slope common across industries, $\kappa = (1 - \vartheta)(1 - \beta\vartheta)/\vartheta$. Price dispersion is second order around the zero-inflation steady state, so that $\hat{\Upsilon}_{s,t} = 0$ in all the first-order conditions of this appendix.

Under the unit elasticities of restriction (viii), the investment price of customer s is the expenditure-share average of its suppliers' prices,

$$\hat{q}^I_{s,t} = \sum_{x=1}^{S} \omega_{I,x\to s}\, \hat{q}_{x,t}. \tag{C.8}$$

With suppliers in rows and customers in columns, these equations imply $\hat{\mathbf{q}}^I_t = \boldsymbol{\omega}'_I \hat{\mathbf{q}}_t$. When the price of a given capital-good supplier goes up, it raises the investment price of all its customer industries. This happens in proportion to that customer industry's expenditure share. However, the investment price of all the industries that purchase capital-good from other suppliers does not need to change. The same restriction makes the component demands (B.14) and (B.17) log-linearize to

$$\hat{C}_{x,t} = \hat{C}_t - \hat{q}_{x,t}, \tag{C.9}$$

$$\hat{I}_{x\to s,t} = \hat{I}_{s,t} + \hat{q}^I_{s,t} - \hat{q}_{x,t}. \tag{C.10}$$

The first allocates aggregate consumption across suppliers, and the second allocates customer s's investment across its capital-goods suppliers.

For the no-intermediate-input analytical economy, log-linearizing goods clearing and

using $s_x^C + \sum_{s=1}^S \psi_{x\to s} = 1$ gives

$$\hat{Z}_{x,t} = s_x^C \hat{C}_t + \sum_{s=1}^{S} \psi_{x\to s}\big(\hat{I}_{s,t} + \hat{q}_{s,t}^I\big) - \hat{q}_{x,t}. \tag{C.11}$$

Supplier x sells to the consumption basket and the investment program of every customer s it equips, each valued at that customer's investment price, and a higher own relative price lowers the quantity demanded from it. Finally, the static factor conditions (B.3)–(B.4), the aggregate production function (B.11) with $\alpha_{H,s} = 0$ and $\hat{\Upsilon}_{s,t} = 0$, and the implied unit cost $\Xi_{s,t} = (W_t/\alpha)^\alpha (R_{s,t}^K/(1-\alpha))^{1-\alpha}$ imply

$$\hat{K}_{s,t} = \hat{Z}_{s,t} + \alpha\hat{w}_t - \alpha\hat{r}_{s,t}^K, \tag{C.12}$$

$$\hat{\Xi}_{s,t} = \hat{w}_t + \frac{1-\alpha}{\alpha}\big(\hat{Z}_{s,t} - \hat{K}_{s,t}\big) - \hat{q}_{s,t}, \tag{C.13}$$

where $\hat{\Xi}_{s,t}$ is the log deviation of sectoral marginal cost relative to the sectoral price. The first condition is the factor demand, in which capital moves one-for-one with scale and responds to the wage–rental ratio. This is modulated by an elasticity which equals the labor share. The second is the unit cost of the input bundle, $\alpha\hat{w}_t + (1-\alpha)\hat{r}_{s,t}^K$. Under the preference restrictions of Section 5.1.1 the real wage equals consumption, and the two conditions are Equation (29) and the marginal-cost definition of Section 6. These are the factor-demand and marginal-cost equations used in the analytical results. When relative prices remain fixed at their steady-state values, as shown in Appendix C.2, the price terms in Equation (C.11) disappear, so that the goods clearing condition becomes Equation (28).

C.2 Relative-Price Inertness

Let $\kappa > 0$ denote the common Phillips slope of restriction (iv); let $\hat{\boldsymbol{\Xi}}_t$ collect sectoral marginal costs relative to sectoral prices and $\boldsymbol{\pi}_t$ sectoral inflation. Restriction (iii) makes Equation (C.13) common across industries, so that marginal cost, the Phillips curve, and the law of motion of relative prices are

$$\hat{\boldsymbol{\Xi}}_t = \hat{C}_t\mathbf{1} + \frac{1-\alpha}{\alpha}(\hat{\mathbf{Z}}_t - \hat{\mathbf{K}}_t) - \hat{\mathbf{q}}_t, \tag{C.14}$$

$$\boldsymbol{\pi}_t = \beta\mathbb{E}_t\boldsymbol{\pi}_{t+1} + \kappa\hat{\boldsymbol{\Xi}}_t, \tag{C.15}$$

$$\hat{\mathbf{q}}_t = \hat{\mathbf{q}}_{t-1} + (\mathsf{I} - \mathbf{1}\boldsymbol{\omega}_C')\boldsymbol{\pi}_t, \qquad \hat{\mathbf{q}}_{-1} = \mathbf{0}. \tag{C.16}$$

Goods-market clearing is now the general Equation (C.11), which carries the relative-price reallocation term; conditional factor demand (29) and the contemporaneous capital law (C.3) are unchanged; and the rental rate is the general user cost of Equation (C.7), which at common depreciation, where $b_s = \bar{b}$, $D_s = \bar{D}$ and $\zeta_s = \zeta$ for every s, and with Equation (C.8) reads

$$\hat{\mathbf{r}}_t^K = \bar{D}\big[\boldsymbol{\omega}_I'\hat{\mathbf{q}}_t - \bar{b}\,\boldsymbol{\omega}_I'\mathbb{E}_t\hat{\mathbf{q}}_{t+1}\big] + \zeta\,\hat{r}_t\mathbf{1}. \tag{C.17}$$

Conditional factor demand gives $\hat{\mathbf{Z}}_t - \hat{\mathbf{K}}_t = -\alpha\mathbf{1}\hat{C}_t + \alpha\hat{\mathbf{r}}_t^K$. Substituting this into

Equation (C.14), the output and capital terms collapse into the rental rate,

$$\hat{\boldsymbol{\Xi}}_t = \alpha \mathbf{1} \hat{C}_t + (1-\alpha)\hat{\mathbf{r}}_t^K - \hat{\mathbf{q}}_t, \tag{C.18}$$

and substituting Equation (C.17) leaves

$$\hat{\boldsymbol{\Xi}}_t = \big[\alpha \hat{C}_t + (1-\alpha)\zeta\, \hat{r}_t\big]\mathbf{1} + (1-\alpha)\bar{D}\big[\boldsymbol{\omega}_I' \hat{\mathbf{q}}_t - \bar{b}\, \boldsymbol{\omega}_I' \mathbb{E}_t \hat{\mathbf{q}}_{t+1}\big] - \hat{\mathbf{q}}_t. \tag{C.19}$$

Because the stock in use at t is installed at t, the intertemporal condition that prices it is dated t, so that it enters the date-t marginal cost directly. Every sector-specific term in Equation (C.19) is a relative price.

Let $\mathsf{M} \equiv \mathsf{I} - \mathbf{1}\boldsymbol{\omega}_C'$ map sectoral inflation into relative prices in Equation (C.16). Since $\boldsymbol{\omega}_C'\mathbf{1} = 1$, we have $\mathsf{M}\mathbf{1} = \mathbf{0}$. Premultiplying Equation (C.19) by M therefore removes the terms common across sectors:

$$\mathsf{M}\hat{\boldsymbol{\Xi}}_t = \mathsf{M}\Big[(1-\alpha)\bar{D}\big(\boldsymbol{\omega}_I' \hat{\mathbf{q}}_t - \bar{b}\, \boldsymbol{\omega}_I' \mathbb{E}_t \hat{\mathbf{q}}_{t+1}\big) - \hat{\mathbf{q}}_t\Big]. \tag{C.20}$$

Premultiplying the Phillips curve (C.15) by M and using $\mathsf{M}\boldsymbol{\pi}_t = \hat{\mathbf{q}}_t - \hat{\mathbf{q}}_{t-1}$ from Equation (C.16) turns Equation (C.20) into

$$\hat{\mathbf{q}}_t - \hat{\mathbf{q}}_{t-1} = \beta \mathbb{E}_t\big(\hat{\mathbf{q}}_{t+1} - \hat{\mathbf{q}}_t\big) + \kappa \mathsf{M}\Big[(1-\alpha)\bar{D}\big(\boldsymbol{\omega}_I' \hat{\mathbf{q}}_t - \bar{b}\, \boldsymbol{\omega}_I' \mathbb{E}_t \hat{\mathbf{q}}_{t+1}\big) - \hat{\mathbf{q}}_t\Big]. \tag{C.21}$$

Equation (C.21) depends only on relative prices, with $\hat{\mathbf{q}}_{-1} = \mathbf{0}$. Hence, $\hat{\mathbf{q}}_t = \mathbf{0}$ for all $t \geq 0$ solves it for every $\kappa > 0$ and every column-stochastic $\boldsymbol{\omega}_I$. Neither consumption nor the real interest rate enters this equation. Equation (C.8) then gives $\hat{\mathbf{q}}_t^I = \boldsymbol{\omega}_I' \hat{\mathbf{q}}_t = \mathbf{0}$.

With $\hat{\mathbf{q}}_t = \mathbf{0}$ the reallocation term drops out of Equation (C.11), which collapses to Equation (28); Equation (C.17) collapses to Equation (31); and marginal cost in Equation (C.19) becomes common across industries, so that sectoral inflation is common as well and relative prices indeed never move. □

With sectoral cost-push shocks. These shocks add $\mathsf{M}\boldsymbol{u}_t$ to Equation (C.21). Relative prices can then move, but the equation still does not depend on consumption or the real interest rate. Thus, monetary policy does not affect relative prices.

C.3 Proof of Proposition 1

Let $\mathsf{L}_I = (\mathsf{I} - \Psi^I)^{-1}$. The adding-up condition $\mathbf{s}^C = \mathbf{1} - \Psi^I\mathbf{1}$ gives the two identities

$$\mathsf{L}_I\Psi^I = \mathsf{L}_I - \mathsf{I}, \qquad \mathsf{L}_I(\mathbf{s}^C + \alpha\Psi^I\mathbf{1}) = (1-\alpha)\mathbf{1} + \alpha\mathbf{U}^I. \tag{C.22}$$

Relative-price inertness, proved in Appendix C.2, implies $\hat{\mathbf{q}}_t^I = \mathbf{0}$. With common depreciation, Equation (C.7) then gives $\hat{\mathbf{r}}_t^K = \zeta\hat{r}_t\mathbf{1}$. The Euler equation, iterated forward with consumption returning to its steady state, gives Equations (26) and (33).

Conditions. We impose the stability condition $\gamma < \delta/(2-\delta)$ stated in Section 5.1.3 and consider interest-rate paths satisfying $\sum_{t\geq 0}(t+1)|\hat{r}_t| < \infty$. Since Ψ^I is similar to $\gamma\boldsymbol{\omega}_I$, every eigenvalue μ of Ψ^I satisfies $|\mu| \leq \gamma$. Combining goods clearing, factor demand, and

the capital law gives

$$(\Psi^I - \delta \mathsf{I})\hat{\mathbf{K}}_t = (1-\delta)\Psi^I \hat{\mathbf{K}}_{t-1} - \delta \mathbf{d}_t, \qquad \mathbf{d}_t = (\mathbf{s}^C + \alpha \mathbf{1})\hat{C}_t - \alpha\zeta \mathbf{1}\hat{r}_t.$$

Every eigenvalue of the capital transition matrix therefore satisfies

$$\left|\frac{(1-\delta)\mu}{\mu - \delta}\right| \leq \frac{(1-\delta)\gamma}{\delta - \gamma} < 1.$$

In this setting, the cumulative responses below are therefore well defined.

Let $\mathcal{Z}$, $\mathcal{I}$, and $\mathcal{K}$ collect the cumulative output, investment, and capital responses. Summing the capital law with capital at its steady state initially and in the limit gives $\mathcal{I} = \mathcal{K}$. Summing goods clearing and conditional factor demand therefore yields

$$\mathcal{Z} = \mathbf{s}^C \mathcal{C} + \Psi^I \mathcal{K},$$
$$\mathcal{K} = \mathcal{Z} + \alpha \mathbf{1}\mathcal{C} - \alpha\zeta \mathbf{1}\mathcal{R}.$$

Eliminating $\mathcal{K}$ and premultiplying by L_I gives

$$\mathcal{Z} = \mathsf{L}_I(\mathbf{s}^C + \alpha \Psi^I \mathbf{1})\mathcal{C} - \alpha\zeta\, \mathsf{L}_I \Psi^I \mathbf{1}\, \mathcal{R}.$$

Equation (C.22) and $\mathsf{L}_I \Psi^I \mathbf{1} = \mathbf{U}^I - \mathbf{1}$ then deliver

$$\mathcal{Z} = \big[\mathbf{1} + \alpha(\mathbf{U}^I - \mathbf{1})\big]\mathcal{C} - \alpha\zeta(\mathbf{U}^I - \mathbf{1})\mathcal{R},$$

which is Equation (41). Finally, $\mathbf{U}^I - \mathbf{1} = \mathsf{L}_I \Psi^I \mathbf{1} \geq \mathbf{0}$. Its xth entry is zero exactly when row x of Ψ^I is zero, since all terms in $\mathsf{L}_I \Psi^I \mathbf{1} = \sum_{k \geq 1} (\Psi^I)^k \mathbf{1}$ are nonnegative. Hence the final-demand loading is at least one and the user-cost loading is nonnegative, with both at their lower bound for a sector that sells no investment goods. □

C.4 Proof of Corollary 1

For part (i), differencing Equation (41) across sectors gives

$$\mathcal{Z}_x - \mathcal{Z}_y = \alpha(U^I_x - U^I_y)(\mathcal{C} - \zeta\mathcal{R}), \tag{C.23}$$

which is Equation (42). For part (ii), the ranking follows from Equation (C.23) whenever $\mathcal{C} - \zeta\mathcal{R} < 0$. A tightening is a path of the real interest rate with $\hat{r}_t \geq 0$ at every horizon and $\hat{r}_0 > 0$, so that $\mathcal{R} > 0$ and, by Equation (33), $\mathcal{C} < 0$. Since $\zeta \geq 0$, both terms of $\mathcal{C} - \zeta\mathcal{R}$ are then negative, and the condition holds.

For part (iii), a uniform investment matrix implies $\Psi^I = (\gamma/S)\mathbf{1}\mathbf{1}'$ and $\mathbf{U}^I = (1-\gamma)^{-1}\mathbf{1}$. Every upstreamness gap in Equation (C.23) is then zero, so all sectors have the same response. □

C.5 Steady-State Network Accounting

At the efficient steady state, the contemporaneous capital Euler equation of Appendix C.1, whose steady state is $\bar{Q}^K_s(1-\bar{b}) = \bar{R}^K_s$, together with $\bar{P}^I_s = \bar{Q}^K_s$, $\bar{R}^K_s \bar{K}_s = (1-\alpha)\bar{P}_s \bar{Z}_s$

and $\bar{I}_s = \delta \bar{K}_s$, implies

$$\frac{\bar{P}_s^I \bar{I}_s}{\bar{P}_s \bar{Z}_s} = \frac{(1-\alpha)\delta}{1-\beta(1-\delta)} = \gamma. \tag{C.24}$$

Goods clearing in value units, divided by aggregate consumption expenditure, is

$$v_x = \omega_{C,x} + \gamma \sum_s \omega_{I,x \to s} v_s. \tag{C.25}$$

Stacking these equations proves Equation (20). The bilateral investment flow from supplier x to customer s has value $\gamma \omega_{I,x \to s} v_s$. Dividing consumption and bilateral investment flows by supplier sales therefore gives

$$s_x^C = \frac{\omega_{C,x}}{v_x}, \qquad \psi_{x \to s} = \frac{\gamma \omega_{I,x \to s} v_s}{v_x}. \tag{C.26}$$

Stacking these identities proves the definition of s_s^C and Equation (21); summing over customers proves Equation (22).

Since $\gamma \in (0,1)$ and $\boldsymbol{\omega}_I$ is column-stochastic, the resolvent in Equation (20) has a positive solution $\mathbf{v}$. The adding-up identity (22) then makes Ψ^I nonnegative with row sums below one, so its Leontief inverse exists.

D Optimal Monetary Policy: Derivations and Proofs

This appendix provides the derivations behind the optimal-policy results in Section 6.1. We first derive the welfare loss and then reduce the Ramsey problem to the aggregate conditions used in the main text. We next prove Proposition 2, which separates price stabilization and user cost, and Proposition 3, which studies how they vary with investment upstreamness.

The sales subsidy of restriction (ix) is financed by a lump-sum tax on the household. In the normative economy, this tax is added to the budget constraint (4), while producer profits are rebated through $\mathcal{D}_t$. Neither term changes the household's optimality conditions.

D.1 Welfare Blocks

Household welfare is $\sum_{t\geq 0}\beta^t[\log C_t - \theta N_t]$. Because $\log C_t$ equals its log deviation $\hat{C}_t$ up to a constant, and labor disutility is linear in the level $N_t = \sum_s \bar{N}_s e^{\hat{N}_{s,t}}$, the second-order expansion of welfare is $\sum_t \beta^t[\hat{C}_t - \theta\bar{N}\sum_s \lambda_s(\hat{N}_{s,t} + \hat{N}_{s,t}^2/2)]$ with $\lambda_s \equiv \bar{N}_s/\bar{N} = v_s/V$ and $\theta\bar{N} = \alpha V$ at the efficient steady state of restriction (ix). The linear terms in $\hat{C}_t$ and $\hat{N}_{s,t}$ are eliminated with second-order expansions of goods clearing, of the Cobb–Douglas aggregators, of the production function, including price dispersion, and of the capital law. Up to terms independent of policy, welfare equals $-\frac{1}{2}\sum_t \beta^t \mathcal{L}_t$.

Price dispersion raises the resources needed to produce sectoral goods. Equation (B.11) implies that producing the sectoral good $Z_{s,t}$ requires $\Upsilon_{s,t}Z_{s,t}$ units of the input bundle. Hence, the cost of price dispersion in sector s, divided by steady-state aggregate consumption expenditure, is $v_s\hat{\Upsilon}_{s,t}$. To second order, Equations (B.7) and (B.10) imply that $\hat{\Upsilon}_{s,t} = \vartheta\hat{\Upsilon}_{s,t-1} + \frac{\epsilon}{2}\frac{\vartheta}{1-\vartheta}\pi_{s,t}^2$. Since there is no price dispersion in the initial steady state, it follows that $\sum_t \beta^t\hat{\Upsilon}_{s,t} = \frac{\epsilon}{2\kappa}\sum_t \beta^t\pi_{s,t}^2$. Thus, in discounted terms, the cost of price dispersion in sector s is $v_s(\epsilon/\kappa)\pi_{s,t}^2/2$, so that

$$\frac{1}{2}\boldsymbol{\pi}_t'\Theta_\pi\boldsymbol{\pi}_t, \qquad \Theta_\pi = \frac{\epsilon}{\kappa}\operatorname{diag}(\mathbf{v}). \tag{D.1}$$

Next, the consumption and investment mixes follow from the Cobb–Douglas demands: $\hat{C}_{x,t} - \hat{C}_t = -\hat{q}_{x,t}$ and $\hat{I}_{x\to s,t} - \hat{I}_{s,t} = \hat{q}^I_{s,t} - \hat{q}_{x,t}$. Consumption has social value one, while customer s's investment expenditure has social value γv_s. These terms are

$$\mathcal{L}_t^P = \frac{1}{2}\sum_{x=1}^S \omega_{C,x}\hat{q}_{x,t}^2 + \frac{\gamma}{2}\sum_{s=1}^S v_s\sum_{x=1}^S \omega_{I,x\to s}(\hat{q}_{x,t} - \hat{q}^I_{s,t})^2. \tag{D.2}$$

The consumption and investment mixes depend on relative prices alone. The remaining terms are $\frac{1}{2}\hat{C}_t^2$ and $\frac{1}{2}\sum_s v_s[\alpha\hat{N}_{s,t}^2 + (1-\alpha)\hat{K}_{s,t}^2 - \hat{Z}_{s,t}^2]$. The first is the consumption gap. Since price dispersion is second order, $\hat{Z}_{s,t} = \alpha\hat{N}_{s,t} + (1-\alpha)\hat{K}_{s,t}$ to first order. The other term then becomes

$$\frac{1}{2}\sum_{s=1}^S v_s\big((1-\alpha)\hat{K}_{s,t}^2 + \alpha\hat{N}_{s,t}^2 - \hat{Z}_{s,t}^2\big) = \frac{\alpha(1-\alpha)}{2}\sum_{s=1}^S v_s\big(\hat{N}_{s,t} - \hat{K}_{s,t}\big)^2. \tag{D.3}$$

This is the factor mix. Together with the consumption gap, price dispersion, and the two mixes, it gives Equation (46).

D.2 Ramsey Implementability and Solution

In the normative economy of Section 6, sectoral cost-push wedges $\boldsymbol{u}_t = \boldsymbol{\varepsilon}_t^u$ enter the Phillips curve. The other optimality and market-clearing conditions are those of Appendix C.1. For the reduction below, we use the relative-price law, the Phillips curve, and the user cost at common depreciation:

$$\hat{\mathbf{q}}_t = \hat{\mathbf{q}}_{t-1} + (\mathsf{I} - \mathbf{1}\boldsymbol{\omega}_C')\boldsymbol{\pi}_t, \tag{D.4}$$

$$\boldsymbol{\pi}_t = \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} + \kappa \hat{\boldsymbol{\Xi}}_t + \boldsymbol{u}_t, \tag{D.5}$$

$$\hat{\mathbf{r}}_t^K = \bar{D}\big[\boldsymbol{\omega}_I' \hat{\mathbf{q}}_t - \bar{b}\boldsymbol{\omega}_I' \mathbb{E}_t \hat{\mathbf{q}}_{t+1}\big] + \zeta\, \hat{r}_t \mathbf{1}. \tag{D.6}$$

Equation (D.6) is Equation (C.7) at common depreciation. Since $\bar{D} = 1 + \zeta$ and $\bar{D}\bar{b} = \zeta$, it can also be written as

$$\hat{r}_{s,t}^K = \hat{q}_{s,t}^I + \zeta\big(\hat{r}_t - \mathbb{E}_t \Delta \hat{q}_{s,t+1}^I\big).$$

The price of investment goods still enters the rental rate when capital fully depreciates. The interest rate and expected capital gains no longer enter. The network identity $\mathbf{v}'\boldsymbol{\omega}_I' = (\mathbf{v}' - \boldsymbol{\omega}_C')/\gamma$, together with $\boldsymbol{\omega}_C'\hat{\mathbf{q}}_t = 0$, implies $\hat{q}_t^I = (1-\gamma)\mathbf{v}'\hat{\mathbf{q}}_t/\gamma$, where $\hat{q}_t^I$ is the average relative price of investment goods in Equation (51). With sectoral cost-push shocks, relative prices follow the equation in Appendix C.2. They do not depend on consumption or the real interest rate.

Reduction to an aggregate problem. Conditional factor demand and the production function give $\hat{N}_{s,t} - \hat{K}_{s,t} = \hat{r}_{s,t}^K - \hat{C}_t$. In Equation (D.6), investment prices determine the sectoral part of the rental rate, while $\zeta\,\hat{r}_t$ is common across sectors. Define $\widehat{\mathbf{g}}_t \equiv \bar{D}[\boldsymbol{\omega}_I'\hat{\mathbf{q}}_t - \bar{b}\boldsymbol{\omega}_I'\mathbb{E}_t\hat{\mathbf{q}}_{t+1}]$ and

$$\chi_t \equiv \frac{\mathbf{v}'\widehat{\mathbf{g}}_t}{V}, \qquad \psi_t \equiv \mathbf{v}'\big(\hat{\mathbf{q}}_t - \hat{\mathbf{q}}_{t-1}\big), \qquad f_t \equiv \kappa(1-\alpha)\boldsymbol{\omega}_C'\widehat{\mathbf{g}}_t + \boldsymbol{\omega}_C'\boldsymbol{u}_t, \tag{D.7}$$

the factor-mix term of Equation (46) becomes $\alpha(1-\alpha)V(\zeta\,\hat{r}_t - \hat{C}_t + \chi_t)^2$, apart from terms that monetary policy cannot affect. Similarly, since $\boldsymbol{\pi}_t = \pi_t\mathbf{1} + (\hat{\mathbf{q}}_t - \hat{\mathbf{q}}_{t-1})$ and $\mathbf{1}'\,\mathrm{diag}(\mathbf{v})\mathbf{1} = V$, the price-dispersion term becomes $(\epsilon/\kappa)V\pi_t^2 + 2(\epsilon/\kappa)\psi_t\pi_t$, apart from terms that depend only on relative prices. The consumption and investment mixes also depend only on relative prices. Together with the consumption gap, these expressions give Equation (47). Finally, premultiplying the Phillips curve (D.5) by $\boldsymbol{\omega}_C'$ and using $\boldsymbol{\omega}_C'\hat{\mathbf{q}}_t = 0$, which holds because $\boldsymbol{\omega}_C'\mathsf{M} = \mathbf{0}$ and $\hat{\mathbf{q}}_{-1} = \mathbf{0}$, gives Equation (49).

D.3 Proof of Proposition 2

Write the constraints of the reduced planner's problem as $\hat{C}_t - \mathbb{E}_t\hat{C}_{t+1} + \hat{r}_t = 0$ and $\pi_t - \beta\mathbb{E}_t\pi_{t+1} - \kappa(\alpha\hat{C}_t + (1-\alpha)\zeta\hat{r}_t) - f_t = 0$. Attach multipliers μ_t and λ_t, respectively,

inside the discounted objective, with $\lambda_{-1} = \mu_{-1} = 0$. The first-order conditions are

$$\hat{C}_t - \alpha(1-\alpha)V(\zeta\hat{r}_t - \hat{C}_t + \chi_t) - \kappa\alpha\lambda_t + \mu_t - \beta^{-1}\mu_{t-1} = 0, \tag{D.8}$$

$$\zeta\alpha(1-\alpha)V(\zeta\hat{r}_t - \hat{C}_t + \chi_t) - \kappa(1-\alpha)\zeta\lambda_t + \mu_t = 0, \tag{D.9}$$

$$\frac{\epsilon}{\kappa}\big(V\pi_t + \psi_t\big) + \lambda_t - \lambda_{t-1} = 0. \tag{D.10}$$

Eliminating μ_t between the first two conditions and substituting $\chi_t = \hat{q}^I_t - \zeta\mathbb{E}_t\Delta\hat{q}^I_{t+1}$ gives

$$(1 + \alpha(1-\alpha)V)\hat{C}_t = \alpha(1-\alpha)V\hat{q}^I_t + \kappa\alpha\lambda_t + \zeta(1-\alpha)\mathcal{B}_t, \tag{D.11}$$

where $\mathcal{B}_t$ is the expression in Equation (52). The initial condition $\mu_{-1} = 0$ makes its lagged bracket zero at $t = 0$. Taking the expected difference between $t+1$ and t, the Euler equation implies $\mathbb{E}_t\Delta\hat{C}_{t+1} = \hat{r}_t$. Equation (D.10) gives $\mathbb{E}_t\Delta\lambda_{t+1} = -(\epsilon/\kappa)\mathbb{E}_t\sum_s v_s\pi_{s,t+1}$. Together, these two relations prove Equation (51). No step divides by ζ, so the representation also holds at $\delta = 1$. □

The full-depreciation benchmark. At the boundary $\delta = 1$, contemporaneous installation and $\phi = 0$ imply $K_{s,t} = I_{s,t}$ and $\hat{r}^K_{s,t} = \hat{q}^I_{s,t}$. Moreover, $\gamma = 1-\alpha$, $V = 1/\alpha$, $\alpha(1-\alpha)V = 1-\alpha$, and $\zeta = 0$. The planner still faces the factor-mix gap $\hat{q}^I_{s,t} - \hat{C}_t$, as the relative price of sectoral investment goods does not coincide sector-by-sector with the real wage. The disappearance of durability therefore does not remove the input-price term from the optimal rate.

To make the comparison with a production network explicit, consider an economy with technology $Z_{s,t} = N^{\alpha}_{s,t}H^{1-\alpha}_{s,t}$ and a Cobb–Douglas intermediate-input bundle with the same supplier weights as the investment bundle. Keep preferences, price setting, consumption shares and shocks unchanged. Relabeling $H_{s,t} = I_{s,t} = K_{s,t}$ makes production, goods clearing and factor demand identical. In particular, $\hat{N}_{s,t} - \hat{H}_{s,t} = \hat{q}^H_{s,t} - \hat{C}_t$. The reduced period loss is

$$\hat{C}^2_t + \frac{\epsilon}{\kappa}V^{IO}\pi^2_t + 2\frac{\epsilon}{\kappa}\psi^{IO}_t\pi_t + (1-\alpha)(\hat{q}^H_t - \hat{C}_t)^2,$$

up to terms independent of policy, where $V^{IO} = 1/\alpha$ and $\hat{q}^H_t = \sum_s(v^{IO}_s/V^{IO})\hat{q}^H_{s,t}$. Its optimal rate is

$$\hat{r}^{*,IO}_t = \frac{(1-\alpha)\mathbb{E}_t\Delta\hat{q}^H_{t+1} - \alpha\epsilon\,\mathbb{E}_t\sum_s v^{IO}_s\pi^{IO}_{s,t+1}}{2-\alpha}. \tag{D.12}$$

This is exactly the price-stabilization component of Equation (51) evaluated at full depreciation. At $\delta < 1$, that component has the same form but uses the social values and equilibrium prices of the durable-capital economy.

D.4 Proof of Proposition 3

Throughout, $\bar{b} = \beta(1-\delta)$, $\bar{D} = 1/(1-\bar{b}) = 1+\zeta$, and ε^u_0 denotes the size of the innovation, common across sectors.

Network accounting. Under restriction (x), Equation (20) gives $v_s = 1/S + \gamma V\omega_{I,s}$, so differences in social value are proportional to differences in supplier weights. With

$\mathbf{a} \equiv (\omega_{I,1}, \ldots, \omega_{I,S})'$ and $\mathsf{D}_v \equiv \mathrm{diag}(\mathbf{v})$, Equation (21) gives $\Psi^I = \gamma \mathsf{D}_v^{-1} \mathbf{a}\mathbf{v}'$, hence $(\Psi^I)^2 = \gamma \Psi^I$ and $\mathsf{L}_I = \mathsf{I} + \Psi^I/(1-\gamma)$. Since the row sums of Ψ^I are $1 - 1/(Sv_s)$, $U_s^I = 1 + [1 - 1/(Sv_s)]/(1-\gamma)$, which is strictly increasing in v_s.

Relative prices. Under restriction (x), the investment price $\hat{q}_{s,t}^I = \sum_x \omega_{I,x} \hat{q}_{x,t}$ is common across customers, and so is the rental rate of Equation (D.6). Marginal costs then differ across sectors only through $-\hat{q}_{s,t}$. Subtracting the consumption-weighted average of Equation (D.5) from its sector-s row, and using Equation (D.4), the relative price of the shocked sector solves

$$(1 + \beta + \kappa)\, h_t - h_{t-1} - \beta h_{t+1} = \varepsilon_0^u \, \mathbb{1}\{t = 0\}, \qquad \hat{\mathbf{q}}_t = h_t \Big(\mathbf{e}_s - \frac{1}{S} \mathbf{1} \Big), \tag{D.13}$$

whose solution is $h_t = h_0 p^t$ with $h_0 = \varepsilon_0^u \, p$ and $h_{-1} = 0$, where $p \in (0,1)$ is the stable root of $\beta p^2 - (1 + \beta + \kappa) p + 1 = 0$. The path h_t does not depend on $\mathbf{a}$.

Forcing terms. Let $z_s \equiv \omega_{I,s} - 1/S$ and $g_t \equiv \bar{D}(h_t - \bar{b} h_{t+1})$. Since $\sum_x v_x \hat{q}_{x,t} = h_t (v_s - V/S) = \gamma V z_s h_t$ and $\hat{q}_{j,t}^I = z_s h_t$ for every customer j, Equations (D.6) and (50) and the definition of ψ_t give

$$\chi_t = z_s g_t, \qquad \psi_t = \gamma V z_s \big(h_t - h_{t-1} \big), \qquad f_t = \frac{\varepsilon_0^u}{S} \mathbb{1}\{t = 0\} + z_s \kappa (1 - \alpha) g_t. \tag{D.14}$$

Since $\bar{b} p < 1$, $g_t = \bar{D} h_0 (1 - \bar{b} p)\, p^t > 0$ at every date.

Differences across shocked sectors. The weights in the planner's problem are the same for every shocked sector, while the terms that depend on relative prices vary with z_s. The responses can therefore be written as

$$\hat{C}_t = \hat{C}_t^{\,0} + z_s c_t, \qquad \pi_t = \pi_t^{\,0} + z_s \eta_t, \qquad \hat{r}_t = \hat{r}_t^{\,0} + z_s (c_{t+1} - c_t),$$

where the superscript 0 denotes the response to the part of the shock common across sectors, ε_0^u / S. The coefficients c_t and η_t do not depend on which sector is hit. The price-stabilization and user-cost terms of Equation (51) then vary with z_s according to

$$P_t = \frac{\alpha(1-\alpha) V (h_{t+1} - h_t) - \alpha \epsilon V [\eta_{t+1} + \gamma (h_{t+1} - h_t)]}{1 + \alpha (1 - \alpha) V}, \tag{D.15}$$

$$U_t = c_{t+1} - c_t - P_t. \tag{D.16}$$

Under restriction (x), $v_s = 1/S + \gamma V \omega_{I,s}$ and $z_s = \omega_{I,s} - 1/S$. Since γV is constant across sectors, the derivatives of the price-stabilization and user-cost components in Equation (53) are $P_0/(\gamma V)$ and $U_0/(\gamma V)$, respectively. Hence, the two derivatives show how optimal policy changes when the shock hits a more investment-upstream sector.

Signs of price stabilization and user cost. Write $a \equiv 1 - \alpha$, $c_\delta \equiv \delta \bar{D}$, and $m \equiv \bar{D} - \zeta p = 1 + \zeta (1 - p)$, where p is the relative-price root above. Then $\gamma = a c_\delta$ and $V = (1 - a c_\delta)^{-1}$. We evaluate the exposure slopes to first order in a at zero. Let x_t, y_t, and ℓ_t be the derivatives of the consumption, inflation, and Phillips-curve-multiplier

slopes with respect to a at that point. Differentiating the planner's conditions gives

$$x_t = \kappa \ell_t + \bar{D} g_t - \frac{\zeta}{\beta} g_{t-1}, \tag{D.17}$$

$$y_t = \beta y_{t+1} + \kappa (x_t + g_t), \tag{D.18}$$

$$\ell_t - \ell_{t-1} = -\frac{\epsilon}{\kappa} \big[y_t + c_\delta (h_t - h_{t-1}) \big], \qquad \ell_{-1} = 0. \tag{D.19}$$

Here $g_t = m h_t$ and $g_{-1} = 0$. Solving this system gives

$$\ell_t = \varepsilon_0^u \left[L p^{t+1} + (F - L) \rho^{t+1} \right], \qquad t \geq 0, \tag{D.20}$$

where $\rho \in (0, 1)$ solves $\beta \rho^2 - (1 + \beta + \epsilon\kappa)\rho + 1 = 0$ and

$$R = m \left(\bar{D} + 1 - \frac{\zeta}{\beta p} \right) - c_\delta, \qquad F = -\frac{\epsilon}{\kappa} \left(\frac{\kappa \zeta m}{\beta} + c_\delta \right), \qquad L = -\frac{\epsilon R}{\kappa(\epsilon - 1)}.$$

Differentiating price stabilization with respect to the capital share at impact gives

$$\begin{aligned} \left. \frac{dP_0}{da} \right|_{a=0} &= \varepsilon_0^u \mathcal{S}_0, \\ \mathcal{S}_0 &= -p(1-p) - \kappa \big[L p (1-p) + (F - L) \rho (1 - \rho) \big]. \end{aligned} \tag{D.21}$$

For user cost, we obtain

$$\left. \frac{dU_0}{da} \right|_{a=0} = \varepsilon_0^u \mathcal{U}_0, \qquad \mathcal{U}_0 = -\zeta p \left[(1-p)(m + 1 - p) + \frac{m}{\beta} \right]. \tag{D.22}$$

For $\delta < 1$, $\zeta > 0$ and hence $\mathcal{U}_0 < 0$. The additional price-stabilization condition is

$$\mathcal{S}_0 > 0. \tag{D.23}$$

Under a sufficiently small positive capital share, these signs imply the two derivatives in Equation (53), since $\gamma V > 0$ and social value and investment upstreamness have the same ranking. At $\delta = 1$, the user-cost term vanishes identically. □