Kinks and Gains from Credit Cycles

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Motivation

- Post-2008: Large focus on financial frictions in macro models, e.g. through collateral constraints on households/firms, following Kiyotaki and Moore (1997).
  - Examples (positive) include Mendoza (2010), Jermann and Quadrini (2012), Liu *et al.* (2013), Guerrieri and Iacoviello (2017), and Jensen *et al.* (2018, 2020).
  - A related literature (normative) uses models of this type to study the implications for optimal macroprudential policy (e.g., Bianchi, 2011; Bianchi and Mendoza, 2018).
- In this paper, we use a small open economy model with an (occasionally binding) collateral constraint to revisit a classic normative question in macroeconomics:
- **What is the welfare cost of business cycles in this context?**
- Main finding: the welfare cost of business cycles is *negative*, i.e., it is a **welfare gain**.

Why can business cycles be beneficial for welfare?

- All else equal, households prefer a stable consumption path to one that fluctuates around the same mean, i.e., the **fluctuations effect** discussed by Lucas (1987).
- In our model, this effect is present, but it is dominated by other forces:
- Occasionally binding credit constraints introduce a *kink* in households' policy and value functions, and thus a **convexification** of their expected continuation value.
- Intuitively, if the household is facing a binding borrowing constraint, business cycles (or uncertainty / shocks) are a potential way to become unconstrained.
- Thus, starting from a second-best situation, uncertainty may be beneficial.
- We show that the gain is not a reflection of pecuniary externalities.

Welfare gains from uncertainty / business cycles: Literature

- Large literature studies the welfare cost of business cycles. Details
- The idea that business cycles may be **beneficial** is rare, but few examples exist:
  - The **Oi-Hartman-Abel effect** (Oi, 1961; Hartman, 1972; Abel, 1983): If profits are convex in demand or costs, higher uncertainty can raise output/profits. Firms expand in response to positive shocks, and scale down after negative ones ("make hay when the sun shines").
    - A household version is studied by **Cho *et al*. (2015)**, who find a welfare *gain* from business cycles in a RBC environment with technology shocks, as households choose to work more when productivity is high, and vice versa.
  - **Chetty and Szeidl (2007)**: When households face consumption commitments (e.g., housing), the value function becomes locally convex at the edges of the (S,s) band, so that households would prefer a lottery ticket over a dollar with certainty (i.e., prefer uncertainty).
  - To our knowledge, no existing studies propose financial frictions as a driver of the gain.

Agenda

- The Model
- Dissecting the welfare gain
- Numerical results
- Extensions

The Model: Environment

- Small open economy assumption (exogenous R)
- Endowment economy with exogenous income process
- Utility over non-durable consumption and housing
- Representative household with discount factor $\beta < \frac{1}{R}$
- Domestic households can borrow abroad, subject to a collateral constraint

The Model

- Utility of the representative household:

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma} c_t^{1-\gamma} + \frac{\nu}{1-\gamma_h} h_t^{1-\gamma_h} \right) \right].$$

- The budget constraint is given by:

$$c_t + q_t(h_t - h_{t-1}) + Rd_{t-1} = yf\left(e_t\right) + d_t,$$

- Household's (international) borrowing is subject to a collateral constraint:

$$d_t \leq (s + s_t) \frac{\mathbb{E}_t q_{t+1} h_t}{R}. \tag{1}$$

The Model

- In the model, business cycles originate from two types of exogenous disturbances:
- **Income shocks**:

$$e_t = \rho_e e_{t-1} + u_t^e, \qquad 0 < \rho_e < 1, \qquad u_t^e \sim \mathrm{N}\left(0, \sigma_e^2\right),$$

- **Credit-limit shocks**:

$$s_t = \rho_s s_{t-1} + u_t^s, \qquad 0 < \rho_s < 1, \qquad u_{t+1}^s \sim \mathrm{N}\left(0, \sigma_s^2\right).$$

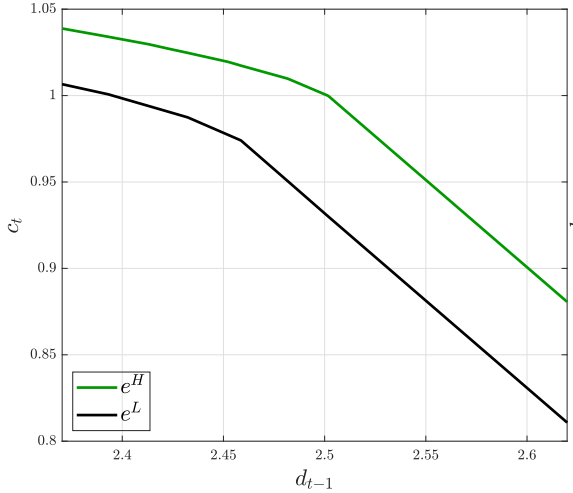
Equilibrium and Solution

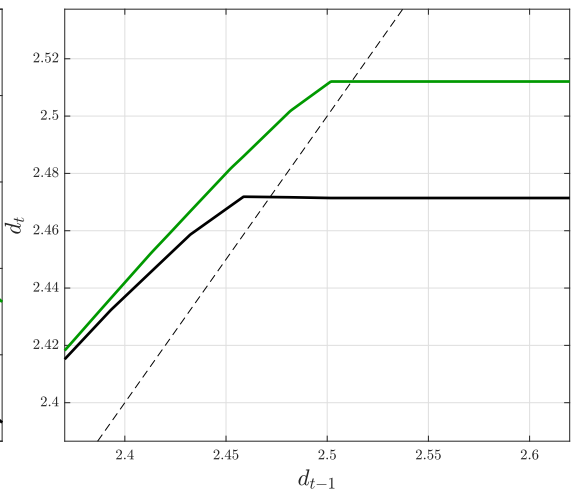
- We assume that housing is in fixed supply; i.e., $h_t = \bar{h}, \forall t$.
- The assumption $\beta < \frac{1}{R}$ implies that the collateral constraint is binding in the economy's steady state, but it may become non-binding in response to shocks.
- We solve the model numerically with global methods. We use the method of Rouwenhorst (1995) to discretize each shock process over a 5-state Markov chain, and then iterate on the Euler equation.

Dissecting the gain

- All else equal, households prefer a stable consumption path to one that fluctuates around the same mean, i.e., the **fluctuations effect** discussed by Lucas (1987).
- Why, then, do households in this model obtain a gain from uncertainty?
- Occasionally binding credit constraints introduce a **kink** in households' policy functions, and thus a **convexification** of their expected continuation value.
- To illustrate the effect of convexification, consider a stylized environment where:
  - We abstract from credit shocks ($s_t = 0$ at all times)
  - Income shocks follow a two-state process; $e_t \in \{e^L, e^H\}$, each with probability ½.

Policy functions of consumption and debt





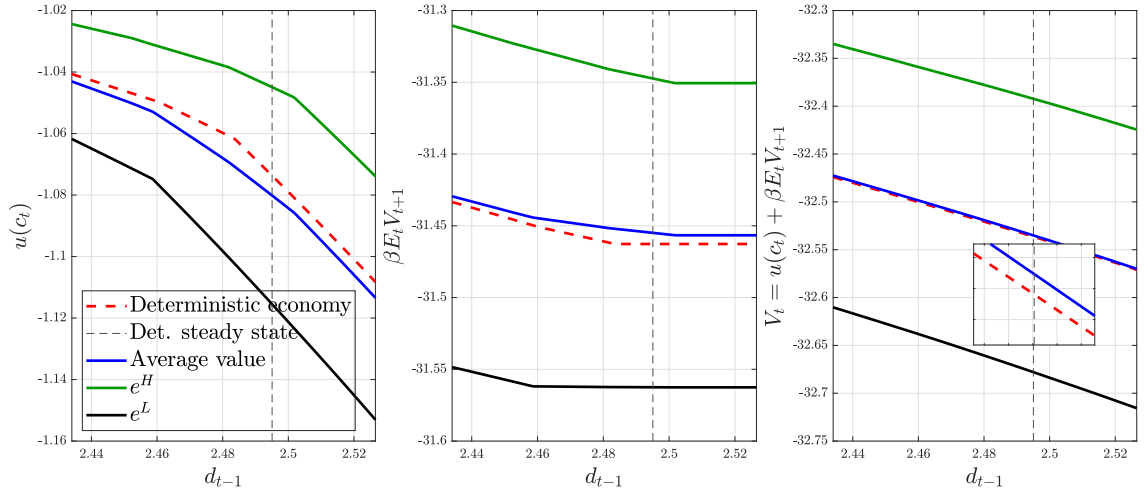
Decomposition of the value function

- The kinked policy function (notably, the disconnect between d_t and d_{t-1}) reflects into the value function, V.
- Recall that $V\left(d_{t-1}, z_t\right)$ has the following recursive representation:

$$V\left(d_{t-1}, z_t\right) = \underbrace{u\left(c\left(d_{t-1}, z_t\right)\right)}_{\text{Contemporaneous utility}} + \underbrace{\beta \mathbb{E}_t V\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)}_{\text{Continuation value}}$$

- We can plot each component of V in this simplified environment.

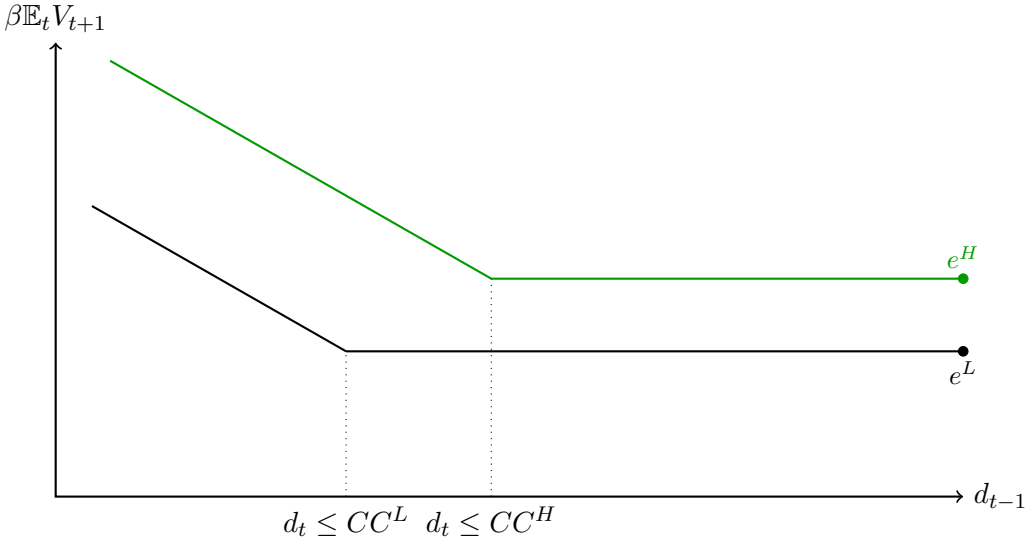
Decomposition of the value function



Decomposition of the value function

- **Left panel**: The kink makes contemporaneous utility *concave*.
  - Negative realization of shocks has a stronger impact on the economy (and welfare) than a corresponding positive shock (as in Mendoza, 2010; or Jensen *et al.*, 2020).
- **Middle panel**: In contrast, the expected continuation value becomes **convex**.
  - Beyond the kink, higher d_{t-1} has no impact on debt at the beginning of period $t+1$ (i.e., d_t).
  - Thus, from $t+1$ onwards, the welfare gain from getting a good shock *in period t* is larger than the loss from getting a bad shock.
- **Right panel**: The value function itself (the sum of the two components) is higher in the stochastic economy (blue line) than in the deterministic one (dashed red line).
  - Uncertainty is welfare-improving!

Convexity of the value function: Mean-preserving spread



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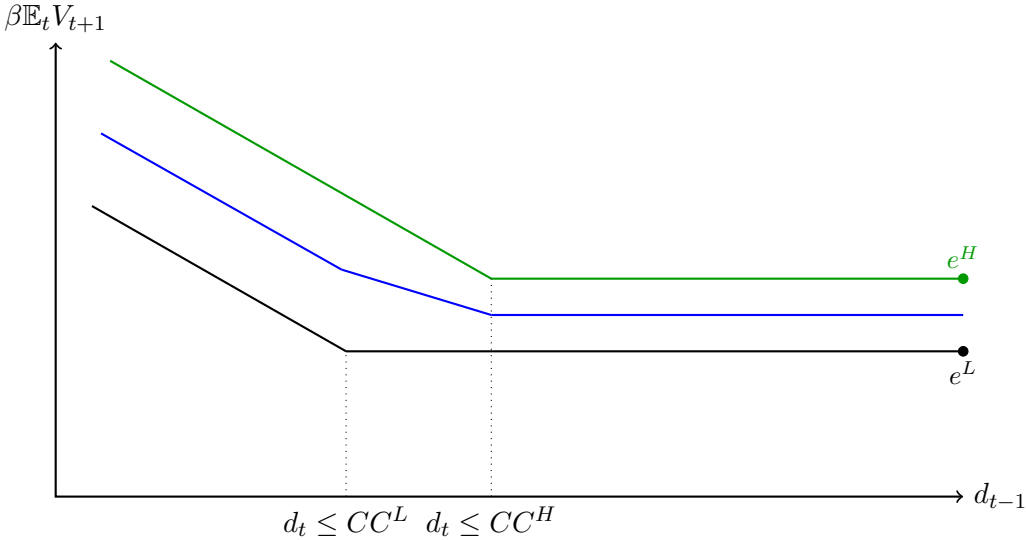

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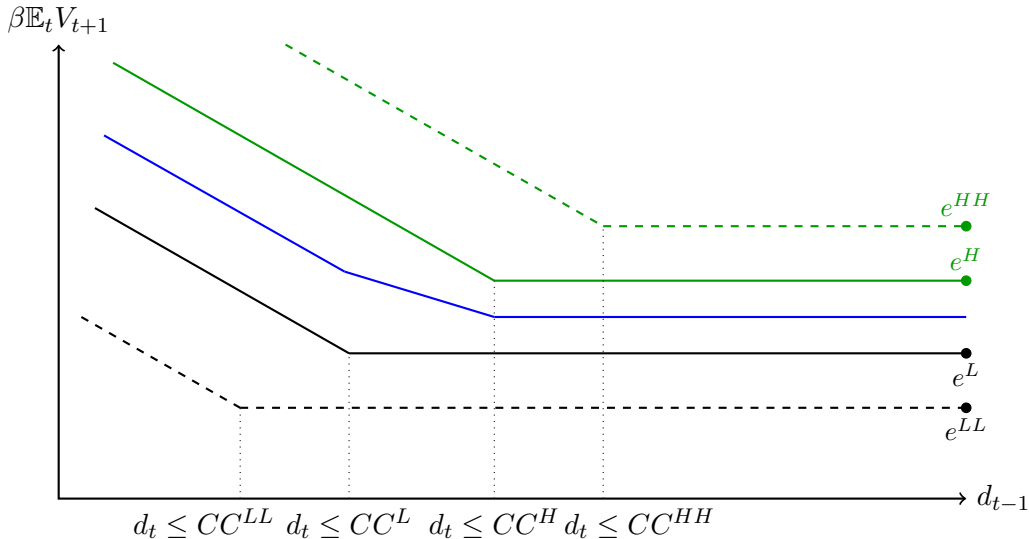

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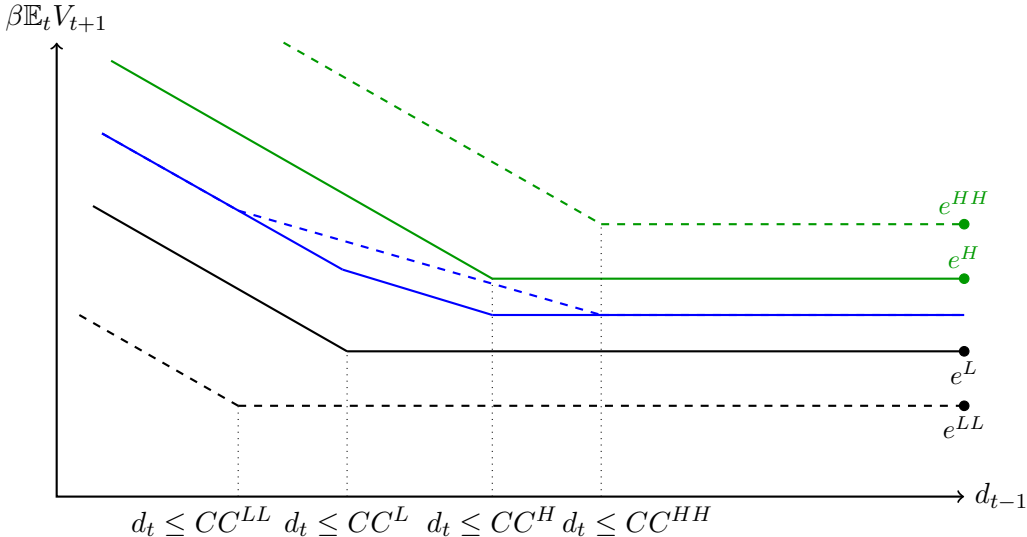


Convexification and welfare gains

- Effectively, the strength of this convexification effect determines whether or not we observe a welfare gain from business cycles.
- To see this, consider another two stylized cases.
  - **Fixed credit limit**: Replace the collateral constraint (1) with a fixed credit limit:

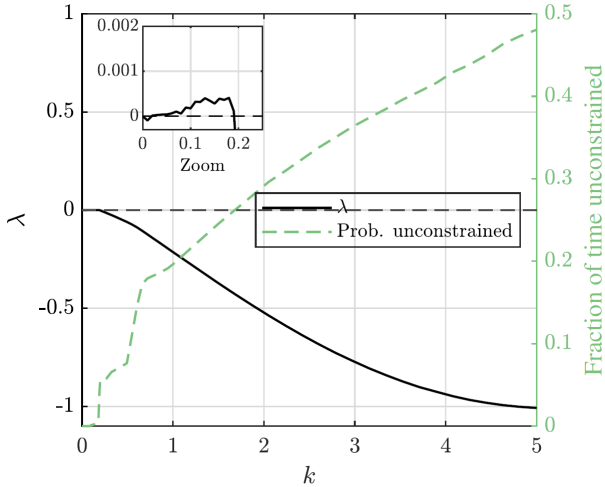
$$d_t \leq s\frac{qh}{R}.$$

    Switches off the convexification effect, as the kinks do not move around horizontally.
  - **Credit-limit shocks**: With shocks to s_t instead of e_t, convexification *strengthens*, and the welfare gain becomes larger (the kinks move around even more).
- Combination of convex continuation value and horizontally shifting kinks is key.

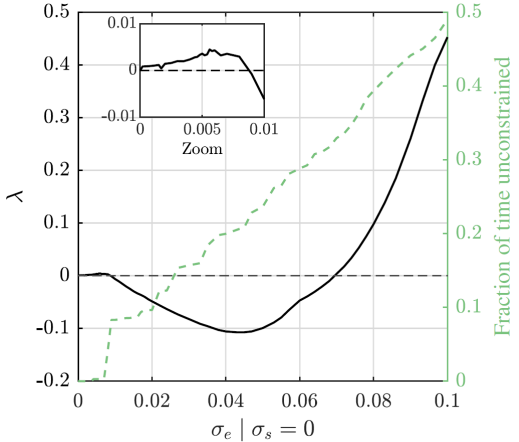
Numerical evidence

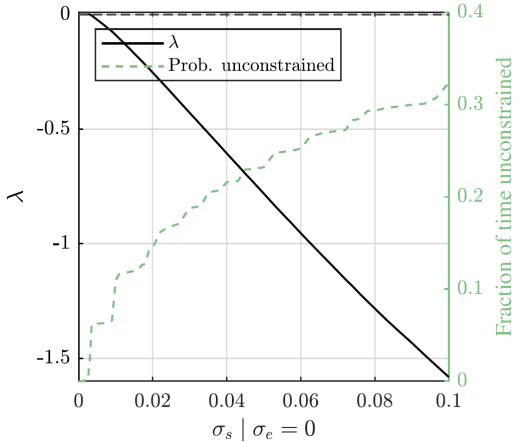
- We turn to model simulations to study the welfare gain from a quantitative viewpoint.
  - Consider the baseline 5-state discretization of the shocks.
  - We first allow for both shocks at once, then one at a time.
- We report the welfare gain as a function of the volatility of the shocks.
  - That is, we multiply the vector of standard deviations $[\sigma_e, \sigma_s]$ by a constant k.
- Our calibration targets the average OECD economy. Details
- Consistent with the exercise of Lucas (1987), we compute the welfare cost of business cycles (λ) as *the percentage increase in stochastic consumption required to obtain the same unconditional welfare as in the economy with no shocks*.
- For the baseline calibration, we obtain $\lambda = -0.2128$, i.e., a **welfare gain**.

Welfare gain and the size of shocks: Two shocks



Welfare gain and the size of each shock





Extensions

- We finally consider four extensions of the model:
  1. Solving a constrained-efficient social planner's problem.
  2. Current-price (instead of expected future price) constraint. Details
  3. An economy with a non-binding borrowing constraint in steady state. Details
  4. Macroprudential policy in the form of a countercyclical LTV ratio. Details

Constrained-efficient social planner's problem

- Could the welfare gain we observe be a result of a **pecuniary externality**?
- An influential strand of the existing literature has studied the role of pecuniary externalities for welfare within models with collateralized borrowing (e.g., Lorenzoni, 2008; Bianchi, 2011).
  - Pecuniary externality: The failure of individual households to internalize the general equilibrium effects of their borrowing decisions on asset prices.
  - This typically leads to "overborrowing".
- Approach: compare welfare in the decentralized equilibrium to the welfare obtained by a constrained-efficient social planner, who internalizes the effect on prices.

Constrained-efficient social planner's problem

- We show analytically (and verify numerically) that the solution obtained in the decentralized equilibrium *coincides* with the constrained-efficient equilibrium.
  - In other words, the decentralized equilibrium is constrained-efficient, and there is no role for the social planner.
  - This result follows that of Ottonello *et al.* (2022) in a related setting.
- Thus, our welfare gain is not driven by pecuniary externalities, and the welfare gain we detect "survives" in settings where a social planner neutralizes such externalities.

Conclusion and policy implications

- We show that business cycles can give rise to a welfare *gain* in small open economy models with collateralized borrowing.
- The gain appears also when a social planner internalizes pecuniary externalities, and irrespective of the timing of the collateral constraint (q_t or $\mathbb{E}_t q_{t+1}$).
- Useful to think carefully about policy implications:
  - Our results echo the findings of some recent studies (e.g., Ottonello *et al*., 2022; Drechsel and Kim, 2024): One should be very careful when using collateral-constraint models for policy analysis. The devil is in the detail!
  - Policies that reduce macroeconomic volatility $\neq$ policies that increase welfare (relatively easy to come up with counterexamples in this model).

The welfare cost of business cycles: Literature

- Lucas (1987): For realistic levels of consumption volatility and risk aversion, the welfare cost of business cycles is a very small number ($\lambda \approx 0.008$).
- A large, subsequent literature has challenged this view, mostly pointing to modifications that would raise this number significantly (though numbers are still relatively small).
- Some important examples include:
  - Incomplete financial markets (Imrohoroglu, 1989; Krusell and Smith, 1999; Krusell *et al*., 2009).
  - Imperfect competition (Galí *et al*., 2007).
  - Non-expected utility functions (Obstfeld, 1994).
  - Endogenous growth (Barlevy, 2004).
  - Disaster risk (Barro, 2006).

Return

The welfare cost of business cycles: Lucas (1987, 2003)

- Let's briefly recall Lucas' argument why the welfare cost of business cycles is very small.
- Consider a household endowed with a stochastic stream of consumption:

$$c_t = Ae^{\mu t}e^{-(1/2)\sigma^2}\epsilon_t,$$

  where $Ae^{\mu t}$ is mean consumption at time t, and $\log(\epsilon_t)$ is a mean zero stochastic process with variance σ^2.
- The household has CRRA utility:

$$U = E_0\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\gamma}}{1-\gamma}.$$

The welfare cost of business cycles: Lucas (1987, 2003)

- Since the utility function is concave, the household would prefer a stable stream of consumption to a volatile stream that fluctuates around the same mean.
- But by how much? By λ such that:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{((1+\lambda)c_t)^{1-\gamma}}{1-\gamma} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(Ae^{\mu t})^{1-\gamma}}{1-\gamma}.$$

- This expression can be solved for λ to obtain:

$$\lambda \approx \frac{1}{2}\gamma\sigma^2.$$

- Since the std. dev. of aggregate consumption (σ) is $0.02 - 0.05$, and conventional values of risk aversion (γ) are below 5, this is inevitably a small number (Lucas: $\lambda \approx 0.008$).

Calibration

- Our calibration targets the average OECD economy.
- We calibrate β such that—given the shocks described below—the model matches the average skewness of consumption growth across OECD countries from 1980:Q1-2019:Q4, which is -0.90. We obtain $\beta = 0.967$.
  - The occasionally binding borrowing constraint is the only source of asymmetry in the model, and is closely tied to β.
- The preference for durables, ν, is set to target a ratio of (annualized) household debt to GDP of 0.63, as in OECD data.
- Remaining household parameters are standard ($\gamma = \gamma_h = 2$, $R = 1.01$, $s = 0.8$).

Calibration

- The income process is calibrated to match the average standard deviation and autocorrelation of GDP across OECD countries during 1980:Q1-2019:Q4 (at business-cycle frequencies).
  - This yields $\sigma_e = 0.015$ and $\rho_e = 0.908$.
- For the financial shock, we rely on U.S. Flow of Funds data, using the series for loan-to-value ratios of households constructed in Jensen *et al.* (2020). We set the shock process to match the business-cycle component of this series.
  - This yields $\sigma_s = 0.016$ and $\rho_s = 0.934$.

Return

Welfare cost of business cycles

- Consistent with Lucas' exercise described above, we ask the question:
  - *By what percentage should the stochastic consumption path be increased to obtain the same unconditional welfare as in the economy with no shocks*?
- In our model, this number is given by:

$$\lambda = 100 \times \left[\left(\frac{\mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right] - u^h}{\mathbb{E}\left[V\left(d_{t-1}, z_t\right)\right] - u^h} \right)^{\frac{1}{1-\gamma}} - 1 \right],$$

  where $\overline{V}\left(d_{t-1}\right)$ is the value of the deterministic economy, and $V\left(d_{t-1}, z_t\right)$ is the value of the stochastic economy with $z_t = (s_t, e_t)$.
- For the baseline calibration, we obtain $\lambda = -0.2128$, i.e., a **welfare gain**.

Alternative timing of the collateral constraint

- As shown by Ottonello *et al.* (2022), the equivalence between the decentralized equilibrium (DE) and the constrained-efficient equilibrium (CEE) obtains when the collateral constraint features the expected future asset price ($\mathbb{E}_t q_{t+1}$), but **not** when the current asset price (q_t) appears.
- We therefore also consider this case, where the collateral constraint (1) becomes:

$$d_t \leq (s + s_t) \frac{q_t h_t}{R}.$$

- We still observe a welfare gain from business cycles in this case, though it is smaller: $\lambda = -0.0162$ in the DE and $\lambda = -0.0464$ in the CEE.

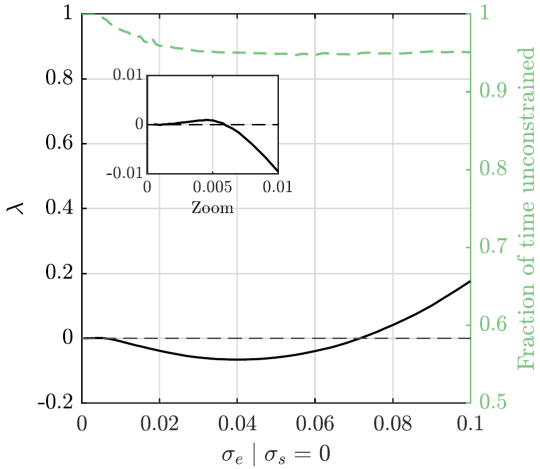
Return

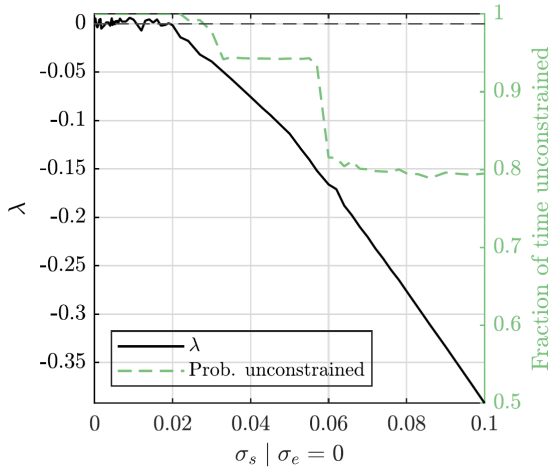
Model with an unconstrained steady state

- While a part of the existing literature has employed models in which the collateral constraint binds in the steady state, another strand of the literature treats the case of an unconstrained steady state (e.g., Mendoza, 2010).
  - Intuitively, one may imagine that in this case, business cycles are less likely to yield a welfare gain.
- When we solve the model under this assumption, results are very similar (although the gain is smaller; $\lambda = -0.0432$).
- Main insights carry over: Convexification effect dominates—requires occasionally binding constraint, "starting point" less important.

Return

Model with an unconstrained steady state





Countercyclical LTV ratio

- We finally use the model to assess the effects of a countercyclical LTV ratio, which is sometimes proposed in macroprudential policy discussions.
- Specifically, the collateral constraint (1) is modified to:

$$d_t \leq \left(s + s_t + \varphi\left(yf\left(e_t\right) - y\right)\right) \frac{\mathbb{E}_t\left[q_{t+1}\right] h_t}{R}.$$

- Thus, $\varphi < 0$ implies a countercyclical LTV ratio (negative income shock $\rightarrow$ higher LTV).

Return

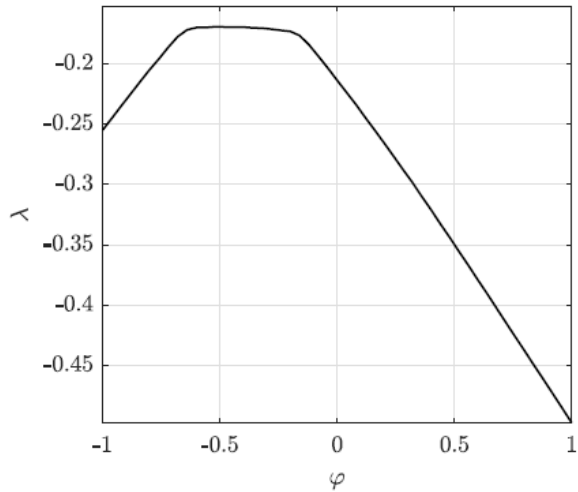
Countercyclical LTV ratio

- In our model, this policy has two effects:
  - *First*, a hedging effect: bad income shocks entail a relaxation of the borrowing constraint.
  - *Second*, the volatility of the LTV ratio is amplified:

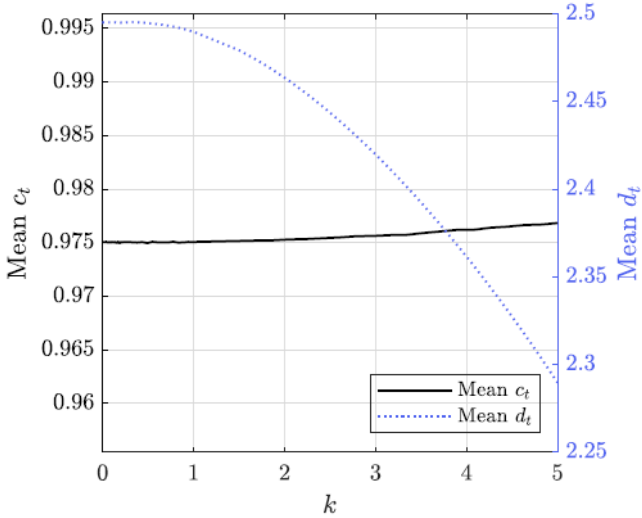
$$\mathbb{V}ar\left(s+\varphi y\left(f\left(e_t\right)-1\right)+s_t\right)=\varphi^2 y^2\left(\exp(\sigma_e^2)-1\right)+\sigma_s^2.$$

- The former effect weakens the convexification channel $\rightarrow$ smaller welfare gain.
  - In addition, households need less precautionary savings $\rightarrow$ higher debt, lower consumption $\rightarrow$ smaller welfare gain.
- The latter effect strengthens the convexification channel $\rightarrow$ larger welfare gain.
  - Note that this effect is quadratic in φ: Small when $\varphi \approx 0$, large when φ is *far from* 0.

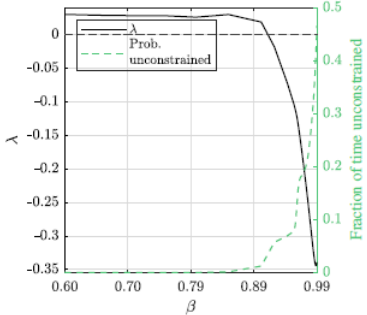
Countercyclical LTV ratio

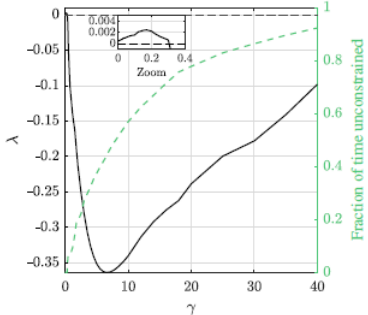


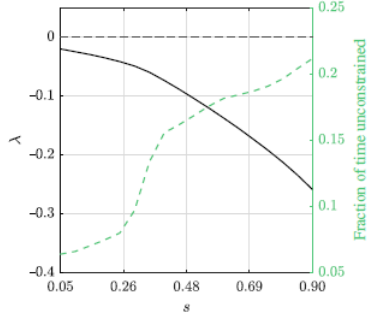
Additional figures: Effect on mean debt and consumption



Additional figures: Robustness checks







Additional figures: Policy functions - baseline model